

SUBJECT – EC-II

BRANCH-ELECTRICAL

SEMESTER-5th SEM

PREPARED BY- PRABHAT RASHMI MALLIK



Lect.(Electrical)

3 ϕ Induction Motor

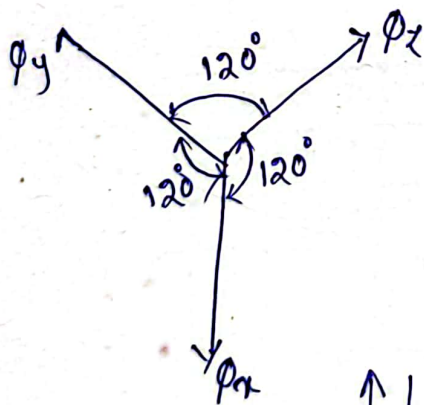
Production of Rotating magnetic Field :-

$\Rightarrow$ When a 3- ϕ winding is energised from 3- ϕ supply, a rotating magnetic field is produced.

This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field.

$\Rightarrow$ It can be shown that magnitude of this rotating field is constant & is equal to $1.5\phi_m$ where $\phi_m = \text{max}^m$ flux due to any phase.

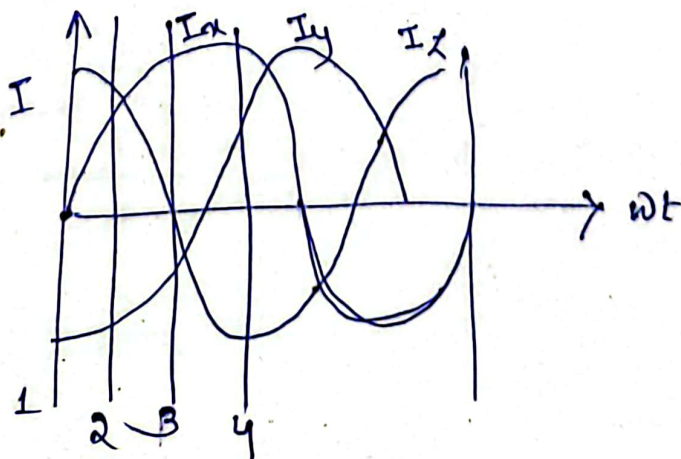
To see how rotating field is produced, consider a 2-pole, 3- ϕ winding as shown in Fig (40). The 3 phases X, Y & Z are energized from a 3- ϕ source & currents in 3 phases are indicated as I_x , I_y & I_z .



$$\phi_x = \phi_m \sin \omega t$$

$$\phi_y = \phi_m \sin (\omega t - 120^\circ)$$

$$\phi_z = \phi_m \sin (\omega t - 240^\circ)$$

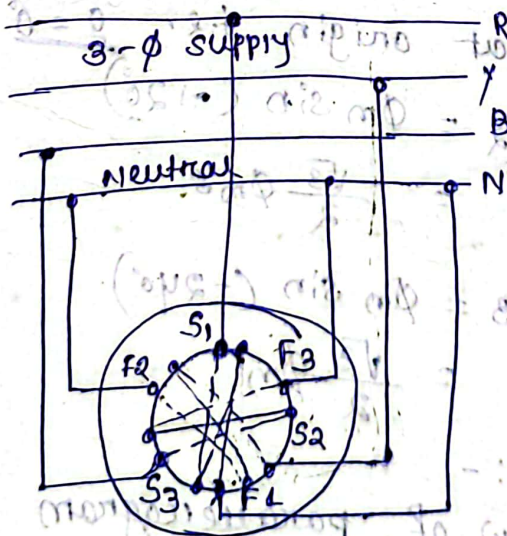


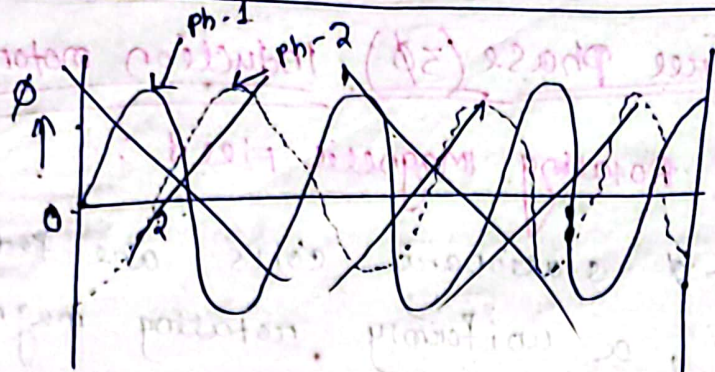
Three Phase (3ϕ) Induction motorProduction of Rotating Magnetic Field :-

$\Rightarrow$ When $3-\phi$ stationary coils are fed with $3-\phi$ supply, a uniformly rotating magnetic flux of constant magnitude will produce.

$\Rightarrow$ It will now be shown that when $3-\phi$ wdg displaced in space by 120° , are fed by $3-\phi$ currents, displaced in time by 120° , they produce a resultant magnetic flux, which rotates in space as if actual magnetic poles were being rotated mechanically.

$\Rightarrow$ The principle of a $3-\phi$, 2 pole stator having 3 identical windings placed 120° space degree apart as shown in Fig (1). The flux due to $3-\phi$ windings is shown in Fig (2).

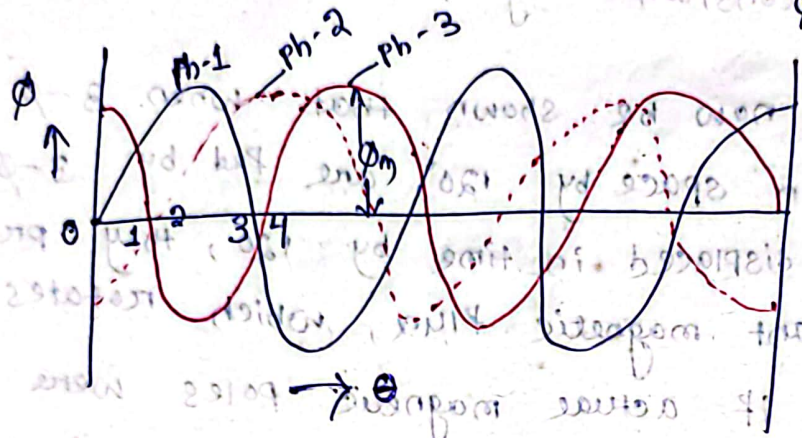




$$\phi_1 = \phi_m \sin \omega t$$

$$\phi_2 = \phi_m \sin(\omega t - 120^\circ)$$

$$\phi_3 = \phi_m \sin(\omega t - 240^\circ)$$



Let the max^m value of Flux due to any one of the 3 phases be ϕ_m . The resultant flux ϕ_r at any instant is given by the vector sum of the individual fluxes ϕ_1, ϕ_2 & ϕ_3 due to 3 phases.

CASE-1

Resultant flux at origin i.e. $\theta = 0^\circ$ At that time $\phi_1 = 0$, $\phi_2 = \phi_m \sin(-120^\circ)$

$$= -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_3 = \phi_m \sin(-240^\circ)$$

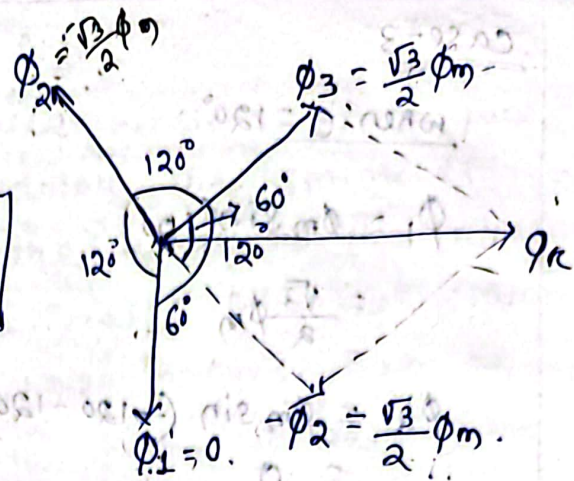
$$= \frac{\sqrt{3}}{2} \phi_m$$

Resultant flux ϕ_r :-

As per law of parallelogram

$$\phi_r^2 = \phi_2^2 + \phi_3^2 + 2\phi_2 \cdot \phi_3 \cdot \cos \theta$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$



$$\Rightarrow \phi_R^2 = \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + 2 \cdot \frac{\sqrt{3}}{2} \phi_m \cdot \frac{\sqrt{3}}{2} \phi_m \cdot \cos 60^\circ$$

$$\Rightarrow \phi_R^2 = \frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2$$

$$\Rightarrow \phi_R^2 = \frac{9}{4} \phi_m^2$$

$$\Rightarrow \phi_R = \frac{3}{2} \phi_m$$

$$\Rightarrow \boxed{\phi_R = 1.5 \phi_m}$$

Case-2

when $\theta = 60^\circ$

Therefore $\phi_1 = \phi_m \sin(60^\circ)$

$$= \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_2 = \phi_m \sin(60^\circ - 120^\circ)$$

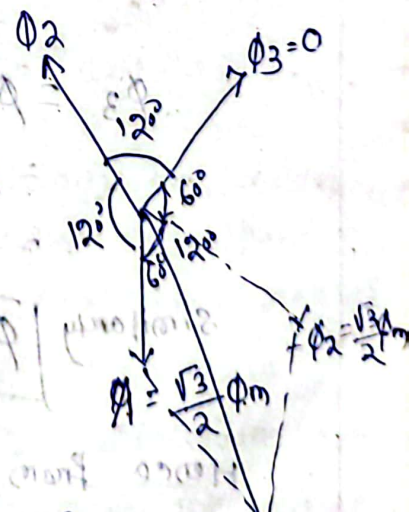
$$= \phi_m \sin(-60^\circ)$$

$$= -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_3 = \phi_m \sin(60^\circ - 240^\circ)$$

$$= \phi_m \sin(-180^\circ)$$

$$= 0$$



$$\phi_R^2 = \phi_1^2 + \phi_2^2 + 2\phi_1 \phi_2 \cos 60^\circ$$

$$\Rightarrow \boxed{\phi_R = 1.5 \phi_m}$$

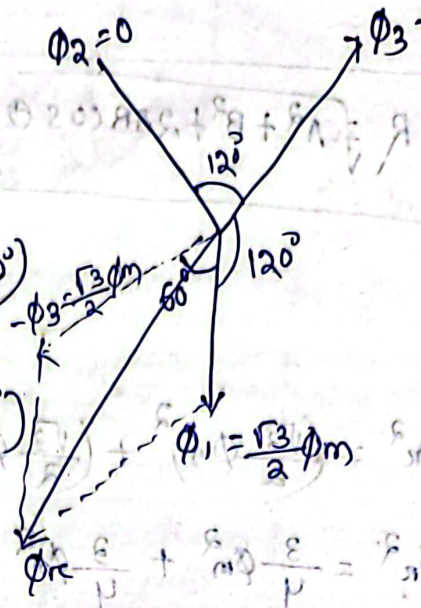
case-3

when $\theta = 120^\circ$

$$\phi_1 = \phi_m \sin(120^\circ) \\ = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_2 = \phi_m \sin(-120^\circ - 120^\circ) \\ = 0$$

$$\phi_3 = \phi_m \sin(120^\circ - 240^\circ) \\ = \phi_m \sin(-120^\circ) \\ = -\frac{\sqrt{3}}{2} \phi_m$$



ϕ_r can be calculated, $\boxed{\phi_r = 1.5 \phi_m}$

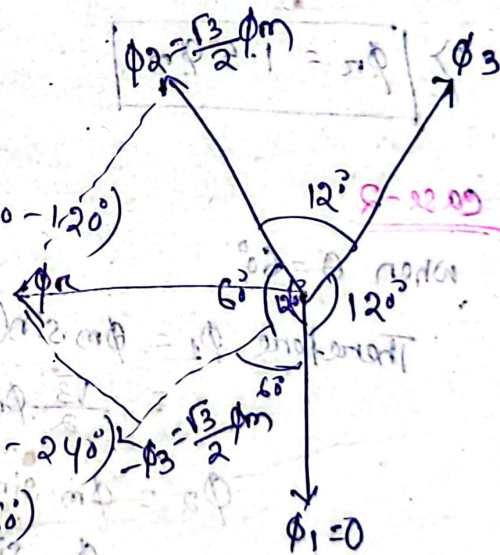
case-4

when $\theta = 180^\circ$

$$\phi_1 = \phi_m \sin 180^\circ \\ = 0$$

$$\phi_2 = \phi_m \sin(180^\circ - 120^\circ) \\ = \phi_m \sin 60^\circ \\ = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_3 = \phi_m \sin(180^\circ - 240^\circ) \\ = \phi_m \sin(-60^\circ) \\ = -\frac{\sqrt{3}}{2} \phi_m$$



Similarly $\boxed{\phi_r = 1.5 \phi_m}$

Hence from the above 4 cases we can draw a conclusion that the resultant flux (ϕ_r) inside the stator winding at any time $= 1.5 \phi_m$ and the resultant flux (ϕ_r) rotates around the stator at synchronous speed.

Construction $\Rightarrow$

$\Rightarrow$ A 3- ϕ IM essentially consists of 2 parts. The stator and the rotor. The stator is the stationary part and the rotor is the rotating part. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

Stator :-

$\Rightarrow$ The stator is built up of high-grade alloy steel laminations to reduce eddy-current losses.

$\Rightarrow$ The laminations are slotted on the inner periphery and are insulated from each other.

These laminations are supported in a stator frame of cast iron or fabricated steel plate.

$\Rightarrow$ The insulated stator conductors are placed in these slots. The stator conductors are connected to form a 3- ϕ winding. The phase winding be either star or delta connected.

$\Rightarrow$ The 3- ϕ stator winding is wound for a definite number of poles as per requirement of speed.

Greater the no. of poles, lesser is the speed of the motor & vice-versa.

$\Rightarrow$ When 3- ϕ supply is given to the stator wdg, a rotating magnetic field of constant magnitude is produced. This rotating field induces current in the rotor by electromagnetic induction.

Rotor :-

→ The rotor mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor wdg) may be one of the following types:-

- (i) squirrel cage rotor
- (ii) wound or slip ring rotor.

(i) Squirrel cage rotor :-

→ It consists of a laminated cylindrical core having parallel slots on its outer periphery.

One copper or aluminium bar is placed in each slot.

→ All these bars are joined at each end by metal rings called end rings. This forms a permanently short-circuited winding which is indestructible.

→ The entire construction (bars and end rings) resembles a squirrel cage and hence the name.

→ The rotor is not connected electrically to the supply but has current induced in it by T/F action from the stator.

→ Those IM which employ squirrel cage rotor are called squirrel cage IM. Most of 3- ϕ IM use squirrel cage rotor as it has a simple and robust construction. However, it suffers from the disadv. of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible

to add any external resistance to the motor ckt to have a large starting torque.

(ii) wound or slip ring rotor :-

⇒ It consists of a laminated core and carries a 3- ϕ wdg similar to the one of the stator.

⇒ The rotor winding is uniformly distributed in the slots and is usually star-connected.

⇒ The open ends of the rotor wdg are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring.

⇒ The 3 brushes are connected to a 3- ϕ star connected rheostat.

⇒ At starting the external resistances are included in the rotor ckt to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.

⇒ The purpose of slip rings and brushes is to provide a means for connecting external resistors in the rotor ckt. The resistors enable the variation of each rotor phase resistance to serve 2 purposes :-

(a) to increase the starting torque and decrease the starting current from the supply.

(b) to control the speed of the motor.

Comparison of cage and wound rotors :-

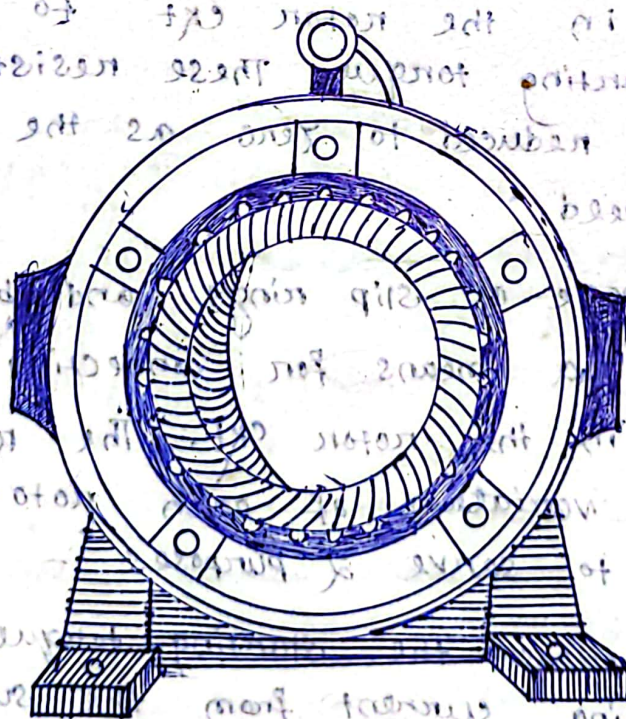
Adv of cage rotor :-

- Robust construction and cheaper.
- Lesser maintenance.
- The absence of brushes reduces the risk of sparking.
- Higher η & higher p.f.

Adv of wound rotor :-

- High starting torque and low starting current.
- Additional resistance can be connected in the rotor ckt to control speed.

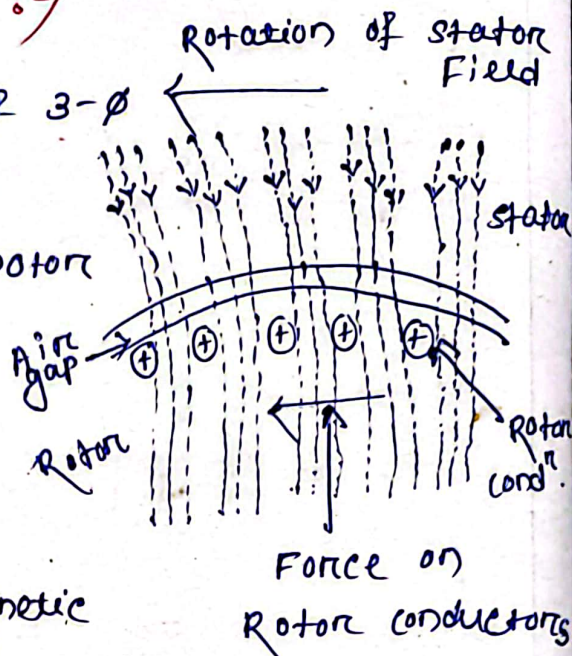
Diagram of stator, rotor [3- ϕ i.m] :-



Principle of operation :-

consider a portion of 3- ϕ l.m. as shown in fig.

The operation of the motor can be explained as under :-



⇒ When 3- ϕ stator wdg. is energised from 3- ϕ supply, a rotating magnetic field is set up which rotates around the stator at synchronous speed ($N_s = \frac{120}{P}$)

⇒ The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, emf's are induced in the rotor conductors. Since the rotor ckt is short-ckted, currents start flowing in the rotor conductors.

⇒ The current carrying rotor conductors are placed in the magnetic field produced by the stator. consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.

⇒ The fact that rotor is urged to follow the stator field (i.e. rotor moves in the direction of stator field) can be explained by ~~Lenz's~~ Lenz's Law.

According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

Slip:-

⇒ The rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux.

⇒ If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor.

⇒ The rotor immediately slow down. Hence, the rotor speed (N) is always less than the stator field speed (N_s).

⇒ The difference between the synchronous speed (N_s) of the rotating stator field and the actual rotor speed (N) is called "slip". It is usually expressed as a % of N_s .

Asynchronous speed diff. of synchronous speed.

$$\% \text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

⇒ The quantity $N_s - N$ called slip speed. $N_s - N$ is the difference between synchronous speed and actual speed.

⇒ When the motor is stationary (i.e. $N=0$) slip, $s = 100\%$.

⇒ In an I.M., the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

~~OR~~ The difference between the synchronous speed and the actual rotor speed is called the "slip speed".

N_s = syn. speed in r.p.m.

N_r = actual rotor speed in r.p.m.

The slip speed = $(N_s - N_r)$ rpm.

The slip speed expressed as a fraction of the syn. speed is called the per-unit slip or fractional slip.

⇒ The per-unit slip is usually called the slip. It is denoted by s .

$$s = \frac{N_s - N_r}{N_s} \text{ (p.u.)}$$

$$\% s = \frac{N_s - N_r}{N_s} \times 100$$

$$\Rightarrow s = \frac{N_s - N_r}{N_s}$$

$$\Rightarrow N_s - N_r = s N_s$$

$$\Rightarrow N_s - s N_s = N_r$$

$$\Rightarrow N_s (1 - s) = N_r$$

$$\Rightarrow \boxed{N_r = N_s (1 - s)} \text{ (rotor speed)}$$

Rotor current frequency :-

The frequency of a voltage or current induced due to the relative speed between a winding and a magnetic field is given by the general

Formula :-
$$F = \frac{NP}{120}$$

where N = Relative speed betⁿ magnetic field and the wdg.

P = No. of poles

For a rotor speed N_r , the relative speed betⁿ the rotating flux and the rotor is $N_s - N_r$. The rotor current frequency f' is given by;

$$f' = \frac{(N_s - N_r) P}{120}$$

$$= \frac{s N_s P}{120}$$

$$= s \cdot f$$

$$\left(\because s = \frac{N_s - N_r}{N_s} \right)$$

$$\left(\because f = \frac{N_s P}{120} \right)$$

i.e. Rotor current frequency = Fractional slip $\times$ supply frequency.

$\Rightarrow$ When the rotor is at standstill or stationary (i.e. $s=1$), the frequency of rotor current is the same as that of supply frequency

$$(f' = sf = 1 \times f = f).$$

$\Rightarrow$ As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

Torque :-

The torque of an I.M is the torque produced at the rotor. Hence $T = T_r$ where T_r is the rotor torque.

In case of D.C. motor torque = Armature Torque = T_a

$$T_a = 0.159 \phi Z I_a \left(\frac{P}{A} \right) \text{ N.m}$$

$$T_a = K \phi I_a \quad \left[\because 0.159, Z, P \text{ \& } A \text{ are constant} \right]$$

where ϕ = flux produced by the field wdg which is pulsating in nature.

Similarly in case of an I.M the torque is also proportional to the product of flux produced in stator and rotor current

However there is another factor which is to be taken i.e. power factor. Because in this case both flux and current are alternating in nature.

Therefore $T \propto \phi I_2 \cos \phi_2$

I_2 = Rotor current (at standstill)

ϕ = Flux produced in the stator.

$\cos \phi_2$ = p.f. of the rotor at standstill.

As $\phi \propto E_2$

$$\therefore T \propto E_2 I_2 \cos \phi_2$$

$$\Rightarrow T = K E_2 I_2 \cos \phi_2$$

($\because E_2$ = rotor emf at standstill)

Starting Torque (T_s):-

Let E_2 = rotor emf / phase at standstill.

X_2 = rotor reactance / phase at standstill.

R_2 = rotor resistance / phase

Rotor impedance / phase, $Z_2 = \sqrt{R_2^2 + X_2^2}$ (at s.s.)

Rotor current / phase, $I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$ (")

Rotor p.f., $\cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$ (")

$$\begin{aligned}
 \therefore \text{Starting torque, } T_s &= K E_2 I_2 \cos \phi_2 \\
 &= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \\
 &= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}
 \end{aligned}$$

Generally, the stator supply voltage V is constant so that flux/pole ϕ set up by the stator is also fixed. This in turn means that e.m.f E_2 induced in the rotor will be constant.

$$\therefore T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 = another constant.

Condition for maximum starting torque :-

$$\text{As starting torque, } T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \quad \text{--- (1)}$$

Differentiating eq (1) w.r.t R_2 and equating the result to zero, we get

$$\begin{aligned}
 \frac{d(T_s)}{dR_2} &= 0 \\
 \Rightarrow \frac{d}{dR_2} \left(\frac{K_1 R_2}{R_2^2 + X_2^2} \right) &= 0 \\
 \Rightarrow K_1 \frac{d}{dR_2} \left(\frac{R_2}{R_2^2 + X_2^2} \right) &= 0
 \end{aligned}$$

$$\Rightarrow \frac{d}{dR_2} \left(\frac{R_2}{R_2^2 + X_2^2} \right) = 0$$

$$\Rightarrow \frac{(R_2^2 + X_2^2) \cdot \frac{d}{dR_2} R_2 - R_2 \frac{d}{dR_2} (R_2^2 + X_2^2)}{(R_2^2 + X_2^2)^2} = 0$$

$$\Rightarrow R_2^2 + X_2^2 \cdot 1 - R_2 (2R_2 + 0) = 0$$

$$\Rightarrow R_2^2 + X_2^2 - 2R_2^2 = 0$$

$$\Rightarrow X_2^2 = R_2^2$$

$$\Rightarrow R_2^2 = X_2^2$$

$$\Rightarrow \boxed{R_2 = X_2}$$

Hence the starting torque will be maximum when

$$\boxed{\text{Rotor Resistance} = \text{Rotor Reactance.}}$$

Torque under running condition :-

Rotor EMF :-

Let E_2 = stand still rotor emf/phase

X_2 = " " reactance/phase

when rotor starts rotating, the relative speed betⁿ rotor and rotating flux in the stator starts decreasing.

$$\text{Slip (s)} = \frac{N_s - N_r}{N_s}$$

The rotor induced emf is directly proportional to this relative speed

$$\begin{aligned} \text{i.e. } E_r &\propto (N_s - N_r) E_2 \\ \Rightarrow E_r &= K (N_s - N_r) E_2 \\ \Rightarrow E_r &= \frac{N_s - N_r}{N_s} \cdot E_2 \\ \Rightarrow E_r &= s \cdot E_2 \end{aligned}$$

Rotor reactance :-

The frequency of the rotor current

$$f_r = s f$$

~~$$X_r = 2\pi f_r L$$~~

$$\therefore X_r = 2\pi f_r L$$

$$\Rightarrow X_r = 2\pi s f L$$

$$X_r = 2\pi s f L$$

$$\Rightarrow X_r = s (2\pi f L)$$

$$\therefore X_r = s \cdot X_2$$

Torque under running condition

Let the rotor at standstill have / phase induced emf E_2 , reactance X_2 and resistance R_2 . Then under running condⁿ at slip s ,

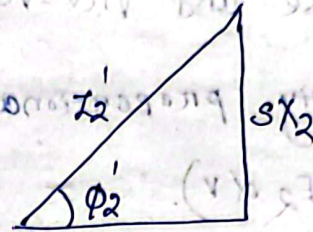
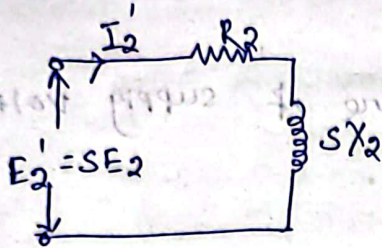
$$\text{Rotor emf / phase, } E_2' = s E_2$$

$$\text{Rotor reactance / phase, } X_2' = s X_2$$

$$\text{Rotor impedance / phase, } Z_2' = \sqrt{R_2^2 + (s X_2)^2}$$

$$\text{Rotor current/phase, } I_2' = \frac{E_2'}{Z_2'} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\text{Rotor p.f. } \cos \phi_2' = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$



(i)

(ii)

Running Torque, $T_r \propto E_2' I_2' \cos \phi_2'$

$$\propto \phi I_2' \cos \phi_2' \quad (\because E_2' \propto \phi)$$

$$\propto \phi \times \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\propto \frac{\phi SE_2 R_2}{R_2^2 + (SX_2)^2}$$

$$= \frac{K \phi SE_2 R_2}{R_2^2 + (SX_2)^2}$$

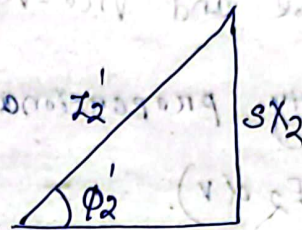
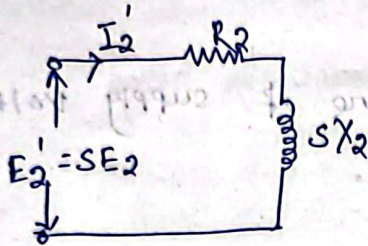
$$T_r = \frac{K \phi SE_2^2 R_2}{R_2^2 + (SX_2)^2} \quad (\because E_2 \propto \phi)$$

If the stator supply voltage V is constant, then stator flux and hence E_2 will be constant.

$$T_r = \frac{K_2 S R_2}{R_2^2 + (SX_2)^2}$$

rotor current/phase, $I_2' = \frac{E_2'}{Z_2'} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}}$

rotor p.f. $\cos \phi_2' = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$



(i)

(ii)

Running Torque, $T_r \propto E_2' I_2' \cos \phi_2'$

$\propto \phi I_2' \cos \phi_2' \quad (\because E_2' \propto \phi)$

$\propto \phi \times \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$

$\propto \frac{\phi SE_2 R_2}{R_2^2 + (SX_2)^2}$

$= \frac{K \phi SE_2 R_2}{R_2^2 + (SX_2)^2}$

$T_r = \frac{K \phi SE_2^2 R_2}{R_2^2 + (SX_2)^2} \quad (\because E_2 \propto \phi)$

If the stator supply voltage V is constant, then stator flux and hence E_2 will be constant.

$T_r = \frac{K_2 S R_2}{R_2^2 + (SX_2)^2}$

Where k_2 = another constant.

It may be seen that running torque is :-

(i) directly proportional to slip i.e. If slip increases (i.e. motor speed decreases), the torque will increase and vice-versa.

(ii) directly proportional to square of supply voltage ($\because E_2 \propto V$).

At starting, $s = 1$ so that starting torque is

$$\left. \begin{aligned} T_r &= 0 \text{ at stand still (and)} \\ s &= \frac{N_s - 0}{N_s} = 1 \end{aligned} \right\}$$

$$T_r = \frac{k_2 E_2^2 R_2}{R_2^2 + X_2^2}$$

Maximum Torque under Running condition :-

$$T_r = \frac{k_2 s R_2}{R_2^2 + s^2 X_2^2} \quad \text{--- (1)}$$

In order to find the value of rotor resistance that gives maximum torque under running condition, differentiate eqn (1) w.r.t 's' and equate the result to zero i.e.

$$\frac{dT_r}{ds} = \frac{k_2 [R_2 (R_2^2 + s^2 X_2^2) - 2s X_2^2 (s R_2)]}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$$\Rightarrow (R_2^2 + s^2 X_2^2) - 2s^2 X_2^2 = 0$$

$$\Rightarrow R_2^2 = s^2 X_2^2$$

$$\Rightarrow \boxed{R_2 = s X_2}$$

Thus for maximum torque (T_m) under running cond's:
 rotor resistance/phase = Fractional slip $\times$ Standstill
 rotor reactance/phase

Now $T_r \propto \frac{s E_2^2}{R_2^2 + s^2 X_2^2}$

For maximum torque, $R_2 = s X_2$ putting $R_2 = s X_2$

in the above expression, the maximum torque T_m is given by;

$$T_m \propto \frac{E_2^2}{2 X_2}$$

$$\Rightarrow \boxed{T_m = \frac{K E_2^2}{2 X_2}}$$

Relation between full load torque & maxⁿ torque:-

AS $T = \frac{K_2 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$

E_2 is practically constant

Hence, $T = \frac{K_2 s R_2}{R_2^2 + (s X_2)^2}$

Therefore, $T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$

Taking full load slip as s_f at full load torque T_f

$T_f \propto \frac{s_f R_2}{R_2^2 + (s_f X_2)^2} \quad \text{--- (1)}$

$$\text{AS } T_{max} = K E_2^2 / 2X_2$$

$$T_{max} \propto \frac{1}{2X_2} \quad \text{--- (1)}$$

$$\frac{(i)}{(ii)} = \frac{T_f}{T_{max}} = \frac{S_f R_2}{R_2^2 + (S_f X_2)^2} \times \frac{2X_2}{1}$$

$$\frac{T_f}{T_{max}} = \frac{2 S_f R_2 X_2}{R_2^2 + (S_f X_2)^2}$$

Dividing X_2^2 on both side

$$\Rightarrow \frac{T_f}{T_{max}} = \frac{2 S_f \frac{R_2}{X_2}}{\frac{R_2^2}{X_2^2} + S_f^2}$$

Taking $R_2/X_2 = a$

$$\Rightarrow \frac{T_f}{T_{max}} = \frac{2 a S_f}{a^2 + S_f^2} = \frac{2 a s}{a^2 + s^2} \quad (\because s = \text{operating slip})$$

In general $\frac{\text{operating torque}}{\text{maximum torque}} = \frac{2 a s}{s^2 + a^2}$

Relation between starting torque & maximum torque :-

$$\text{AS } T_{st} = K \frac{R_2}{R_2^2 + X_2^2}$$

$$\Rightarrow T_{st} \propto \frac{R_2}{R_2^2 + X_2^2} \quad \text{--- (1)}$$

$$\text{but } T_{max} \propto \frac{1}{2X_2} \quad \text{--- (2)}$$

$$\frac{(i)}{(ii)} = \frac{T_{st}}{T_{max}} = \frac{R_2}{R_2^2 + X_2^2} \times \frac{2X_2}{1}$$

$$\Rightarrow \frac{T_{st}}{T_{max}} = \frac{2 R_2 X_2}{R_2^2 + X_2^2}$$

$$\Rightarrow \frac{T_{st}}{T_{max}} = \frac{2 R_2 X_2 / X_2^2}{R_2^2 / X_2^2 + 1} \quad (\text{Dividing } X_2^2 \text{ in both side})$$

$$\Rightarrow \frac{T_{st}}{T_{max}} = \frac{2 \frac{R_2}{X_2}}{\left(\frac{R_2}{X_2}\right)^2 + 1}$$

Taking $\frac{R_2}{X_2} = a$

$$\Rightarrow \boxed{\frac{T_{st}}{T_{max}} = \frac{2a}{a^2 + 1}}$$

Torque - slip characteristics $\Rightarrow$

The motor torque under running condⁿ is given by

$$T = \frac{k_a s R_2}{R_2^2 + s^2 X_2^2}$$

$\Rightarrow$ If a curve is drawn between the torque and slip for a particular value of rotor resistance R_2 , the graph thus obtained is called torque-slip char.

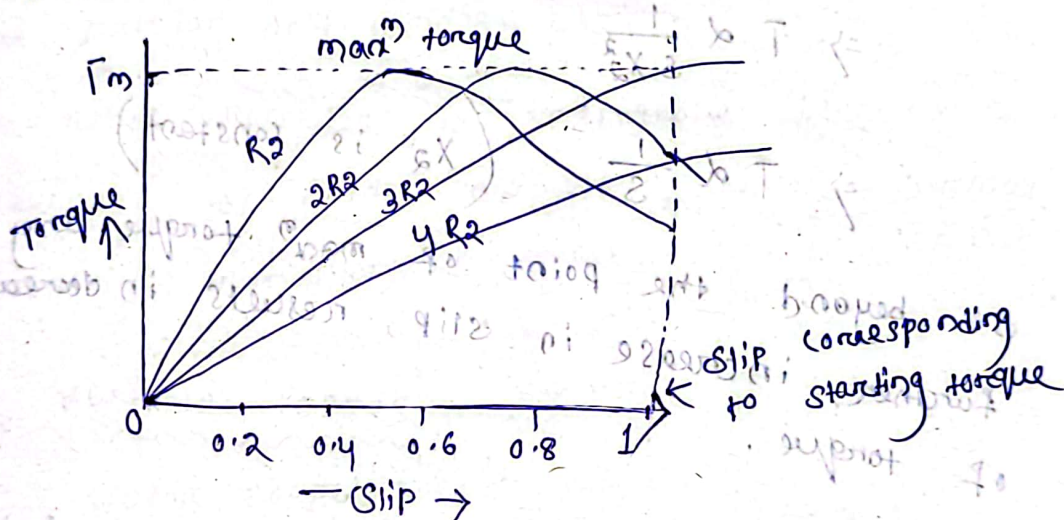


Fig shows torque-slip char. For a slip-range from $s=0$ to $s=1$ for various values of rotor resistance.

$\Rightarrow$ At $s=0$, $T=0$, so that torque-slip curve starts from the origin.

$\Rightarrow$ At normal speed, slip is small so that sX_2 is negligible as compared to R_2 .

$$\therefore T \propto s/R_2$$

$$T \propto s$$

(as R_2 is constant)

Hence torque-slip curve is a straight line from zero slip to a slip that corresponds to full-load.

⇒ As slip increases the torque also increases

it becomes max when $R_2 = sX_2$

$$\text{i.e. } s \pm \frac{R_2}{X_2}$$

As slip further increases (sX_2) becomes higher

compare to R_2 .

Hence R_{rotor} can be neglected in compare to (sX_2)

$$\Rightarrow T \propto \frac{s}{(sX_2)^2}$$

$$\Rightarrow T \propto \frac{1}{sX_2^2}$$

$$\Rightarrow T \propto \frac{1}{s} \quad (X_2 \text{ is constant})$$

So beyond the point of max torque any further increase in slip, results in decrease of torque.

⇒ The max torque remains the same and is independent of the value of rotor resistance.

Therefore, the addition of resistance to the rotor ckt does not change the value of max torque but it only changes the value of slip at which max torque occurs.

Power stages in an induction motor :-

⇒ The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are -

1. Fixed Losses

(i) stator iron loss

(ii) Friction and windage loss

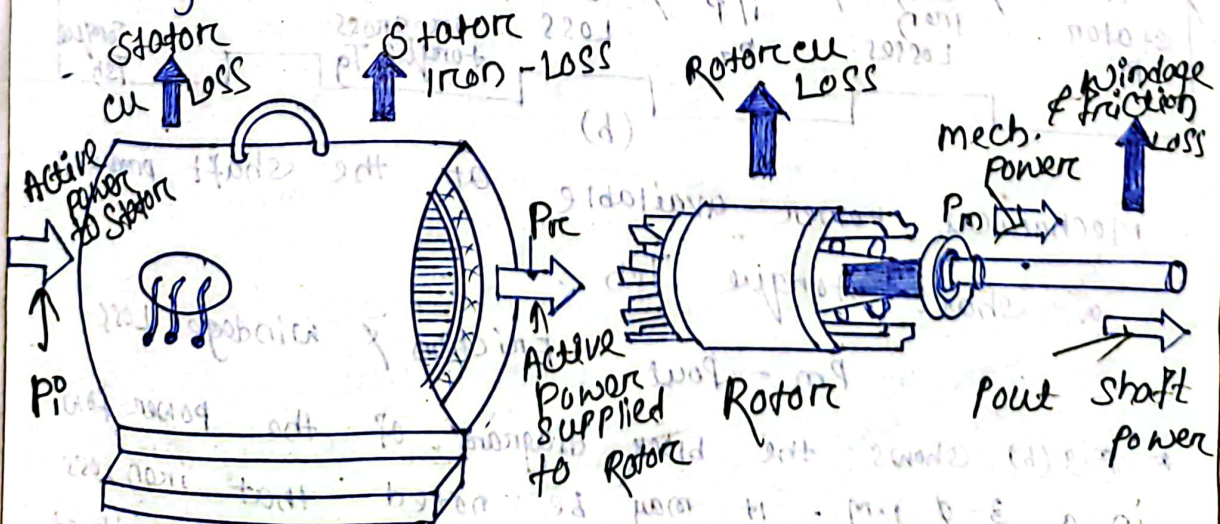
The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

2. Variable losses

(i) stator copper loss

(ii) Rotor copper loss.

Fig. shows how electric power fed to the stator of an i.m. suffers losses and finally converted into mechanical power.



The following points may be noted from the above diagram :-

(i) Stator input, $P_i = \text{stator o/p} + \text{stator losses}$
 $= \text{stator o/p} + \text{stator iron loss} + \text{stator cu loss}$

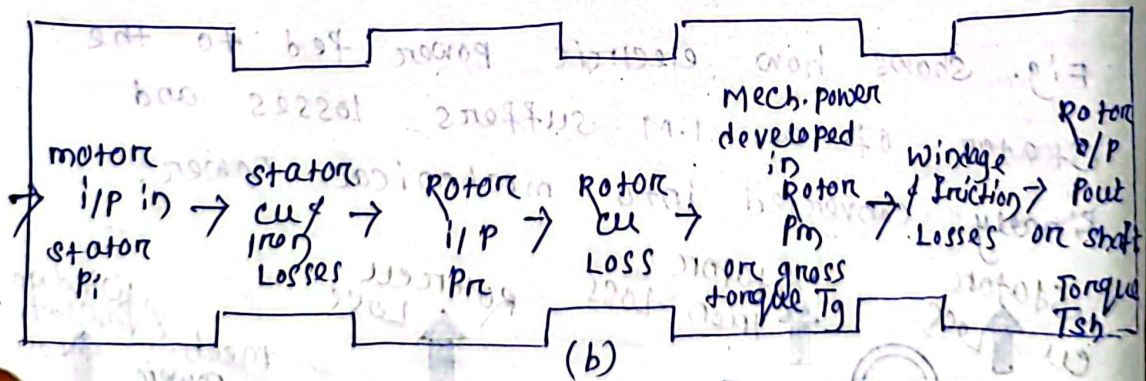
(ii) Rotor input, $P_r = \text{stator o/p}$

It is because stator o/p is entirely transferred to the rotor through air-gap by electromagnetic induction.

(iii) Mechanical power available, $P_m = P_r - \text{rotor cu loss}$

This mechanical power available is the gross rotor o/p and will produce a gross torque T_g .

(iv) Mechanical power at shaft, $P_{out} = P_m - \text{Friction \& Windage Loss}$



Mechanical power available at the shaft produces a shaft torque T_{sh} .

$$P_m - P_{out} = \text{Friction \& Windage Loss}$$

* Fig (b) shows the block diagram of the power flow in a 3- ϕ I.M. It may be noted that iron loss of the rotor is negligible and may be considered

zero. It is because under normal running conditions, the frequency of rotor currents is very small.

Induction motor torque :-

The mechanical power P available from any electric motor can be expressed as :-

$$P = \frac{2\pi NT}{60} \text{ watts}$$

Where N = speed of the motor in rpm

T = Torque developed in N-m

$$T = \frac{60}{2\pi} \frac{P}{N} = 9.55 \frac{P}{N} \text{ N-m.}$$

If (the) gross o/p of the rotor of an i.m. is P_m and its speed is N rpm. Then

$$\text{gross torque } T_g = 9.55 \frac{P_m}{N} \text{ N-m}$$

$$\text{Similarly, } T_{sh} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$

Rotor o/p :-

If T_g N-m is the gross torque developed and N rpm is the speed of the rotor, then

$$\text{Gross rotor o/p} = \frac{2\pi NT_g}{60} \text{ watts}$$

If there were no copper losses in the rotor, the o/p would equal rotor i/p and the rotor would run at syn. speed N_s .

$$\therefore \text{Rotor i/p} = \frac{2\pi N_s T_g}{60} \text{ watts}$$

$$\therefore \text{Rotor cu Loss} = \text{Rotor i/p} - \text{Rotor o/p}$$

$$= \frac{2\pi T_g}{60} (N_s - N)$$

$$(i) \frac{\text{Rotor cu Loss}}{\text{Rotor i/p}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \text{Rotor cu Loss} = s \times \text{Rotor i/p}$$

$$(ii) \text{Gross Rotor o/p, } P_m = \text{Rotor i/p} - \text{Rotor cu Loss}$$

$$P_m = \text{Rotor i/p} - s \times \text{Rotor i/p}$$

$$P_m = \text{Rotor i/p} (1-s)$$

$$(iii) \frac{\text{Gross Rotor o/p}}{\text{Rotor i/p}} = 1-s = \frac{N}{N_s}$$

$$= 1 - \frac{(N_s - N)}{N_s} = \frac{N_s - N_s + N}{N_s} = \frac{N}{N_s}$$

$$(iv) \frac{\text{Rotor cu Loss}}{\text{Gross Rotor o/p}} = \frac{s}{1-s} = \frac{s \times \text{Rotor i/p}}{(1-s) \times \text{Rotor i/p}}$$

$$= \frac{s}{1-s}$$

$$(1-s) \times \text{Rotor i/p} = \text{Gross Rotor o/p}$$

Induction motor Torque Equation :-

The gross torque T_g developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor i/p}}{2\pi N_s} \quad \text{NS in r.p.s.}$$

$$= \frac{\text{Rotor i/p}}{2\pi N_s/60} \quad \text{NS in r.p.m.}$$

$$= \frac{60 \times \text{Rotor i/p}}{2\pi N_s} \quad \text{NS in r.p.m.}$$

Now, Rotor input = Rotor cu loss = $\frac{3(I_2')^2 R_2}{s}$ (1)

under running conditions,

$$I_2' = \frac{SE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{SK E_1}{\sqrt{R_2^2 + (sX_2)^2}}$$

where $K = \text{Transformation Ratio} = \frac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \text{Rotor input} = \frac{3 \times s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{1}{s} = \frac{3s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\text{Also Rotor input} = \frac{3 \times s^2 K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{1}{s} = \frac{3s K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore T_g = \frac{\text{Rotor i/p}}{2\pi N_s} = \frac{3 \times \text{Rotor i/p}}{2\pi N_s} \times \frac{SE_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{(in terms of } E_2)$$

$$= \frac{3 \times \text{Rotor i/p}}{2\pi N_s} \times \frac{SK^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{(in terms of } E_1)$$

Methods of starting different types of starters:

Starting of 3- ϕ I.M :-

⇒ The induction motor is fundamentally a T/F in which the stator is the primary and the rotor is short-circuited secondary.

⇒ At starting, the voltage induced in the I.M is max^m ($\because s=1$). Since the rotor impedance is low, the rotor current is excessively large.

⇒ The large rotor current is reflected in the stator because of T/F action. This results in high starting current (4 to 10 times the full-load current) in the stator at low p.f and consequently the value of starting torque is low.

⇒ Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally. However, this large starting current will produce large line voltage drop. This will affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the starting current and several methods are available for this purpose.

Methods of starting 3- ϕ I.M :-

The method to be employed in starting a given I.M depends upon the size of the motor and the type of the motor. The common methods used to start I.M are :-

- (1) Direct-on-line starting
- (2) Stator Resistance starting
- (3) Auto T/F starting
- (4) Star-Delta starting
- (5) Rotor Resistance starting.

→ Methods (1 to 4) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors.

→ In practice, any one of the 1st 4 methods is used for starting squirrel-cage motors, depending upon the size of the motor. But slip ring motors are invariably started by Rotor Resistance starting.

Methods of starting squirrel-cage motors :-

(1) D-O-L starting :- This method of starting is just what the name implies - the motor is started by connecting it directly to 3- ϕ supply.

→ The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low P.F. This method of starting is suitable for relatively small (upto 7.5 kW) m/c's.

Relation between starting and F.L. torque,

$$\text{Rotor i/p} = 2\pi N_s T = \frac{3}{K} T \quad - N_s \text{ in rps}$$

$$\text{But Rotor cu Loss} = s \times \text{Rotor i/p}$$

$$\Rightarrow \frac{1}{s} (I_2')^2 R_2 = s \times \frac{1}{R_2} \quad (1)$$

$$T \propto (I_2')^2 \quad (2)$$

$$\Rightarrow T = \frac{K (I_2')^2 R_2}{s} \quad (3)$$

$$\Rightarrow T \propto \frac{(I_2')^2}{s} \quad (\text{For same } R_2) \quad (4)$$

$$\Rightarrow T \propto \frac{I^2}{s} \quad (\because I_2' \propto I) \quad (5)$$

If I_{st} is the starting current, then starting torque (T_{st}) is

$$T_{st} \propto I_{st}^2 \quad (\because \text{at starting } s=1) \quad (6)$$

If I_f = Full-load current

s_f = Full-load slip, then

$$\therefore T_f \propto \frac{I_f^2}{s_f} \quad (7)$$

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f} \right)^2 \times s_f \quad (8)$$

When the motor is started D-O-L, the starting current is the short-circuited (blocked-rotor) current I_{sc} .

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f} \right)^2 \times s_f \quad (9)$$

Let us consider the above relation with a numerical example, Let $I_{sc} = 5 I_f$ & Full-load slip $s_f = 0.04$.

Then,
$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f} \right)^2 \times \frac{s_f}{s_r}$$

$$= \left(\frac{5 I_f}{I_f} \right)^2 \times 0.04$$

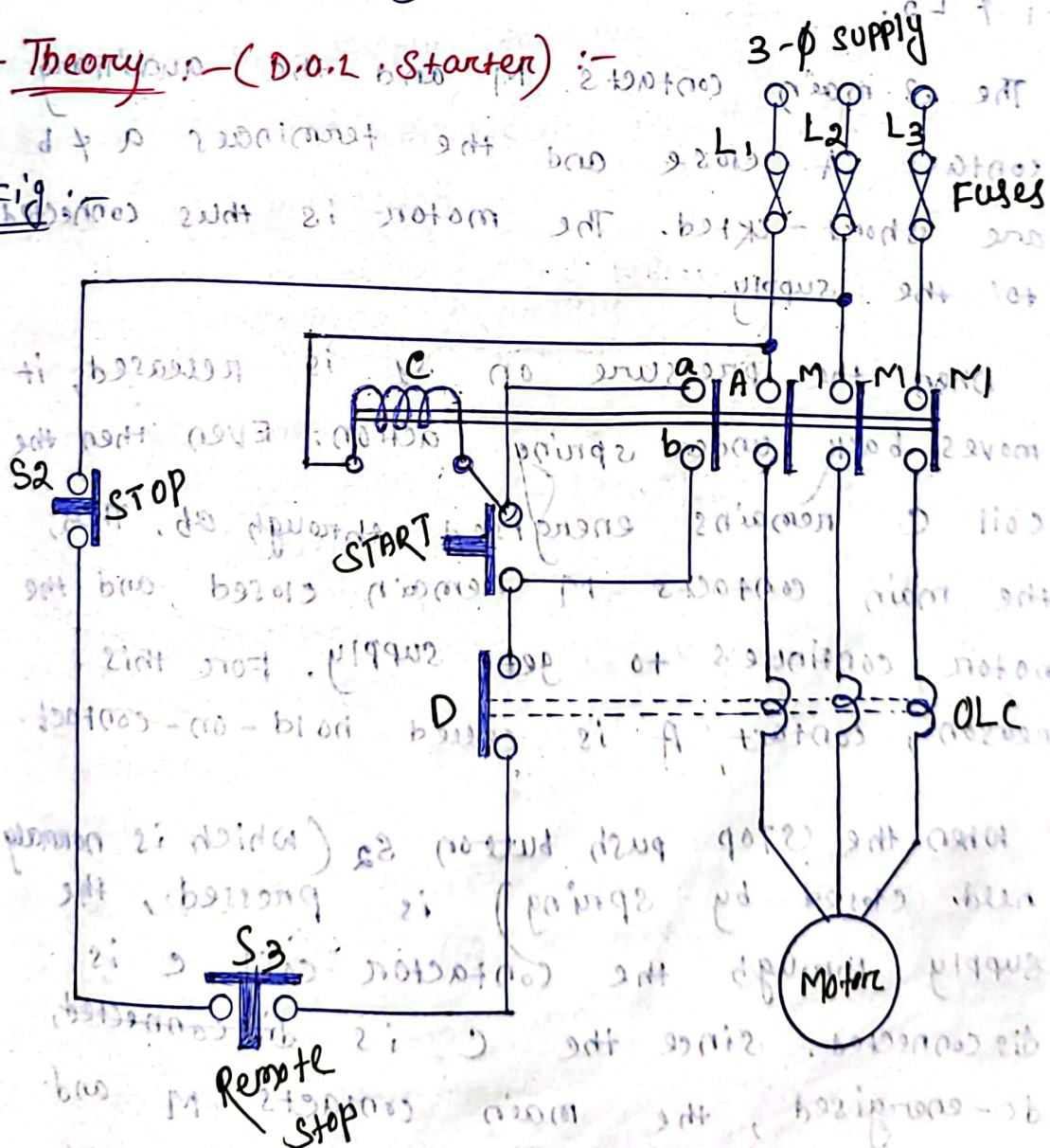
$$= 1$$

∴ $T_{st} = T_f$

Note that starting current is 5 times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low.

OR Theory (D.O.L Starter) :-

Fig:



=> In the D.O.L method of starting cage motors, the motor is connected by means of a starter across the full supply voltage.

=> Fig (1) shows the connections for one type of the D.O.L starter. It consists of a coil-operated contactor C controlled by start and stop push buttons which may be installed at convenient places remote from the starter.

=> On pressing the START push button S_1 (which is normally held open by a spring) the contactor coil C is energised from 2 line conductors

L_1 & L_2 .

=> The 3 main contacts (M_1 and the auxiliary contact A) close and the terminals a & b are short-circuited. The motor is thus connected to the supply.

=> When the pressure on S_1 is released, it moves back under spring action. Even then the coil C remains energised through ab . Thus, the main contacts M remain closed and the motor continues to get supply. For this reason, contact A is called hold-on-contact.

=> When the STOP push button S_2 (which is normally held closed by spring) is pressed, the supply through the contactor coil C is disconnected. Since the C is disconnected, de-energised, the main contacts M and auxiliary contact A are opened. The supply

to motor is disconnected and the motor stops.

Under Voltage Protection :-

When the voltage falls below a certain value, or in the event of failure of supply during motor operation, the coil C is de-energised. The motor is then disconnected from the supply.

Overload Protection :-

In case of an overload on the motor, one or all the overload coils (O.L.C) are energised. The normally closed contact D is opened and the contactor coil C is de-energised to disconnect the supply to the motor.

Fuses are provided in the ckt for short - ckt protection.

(2) Stator - Resistance Starting :- 7/T 02WA (S)

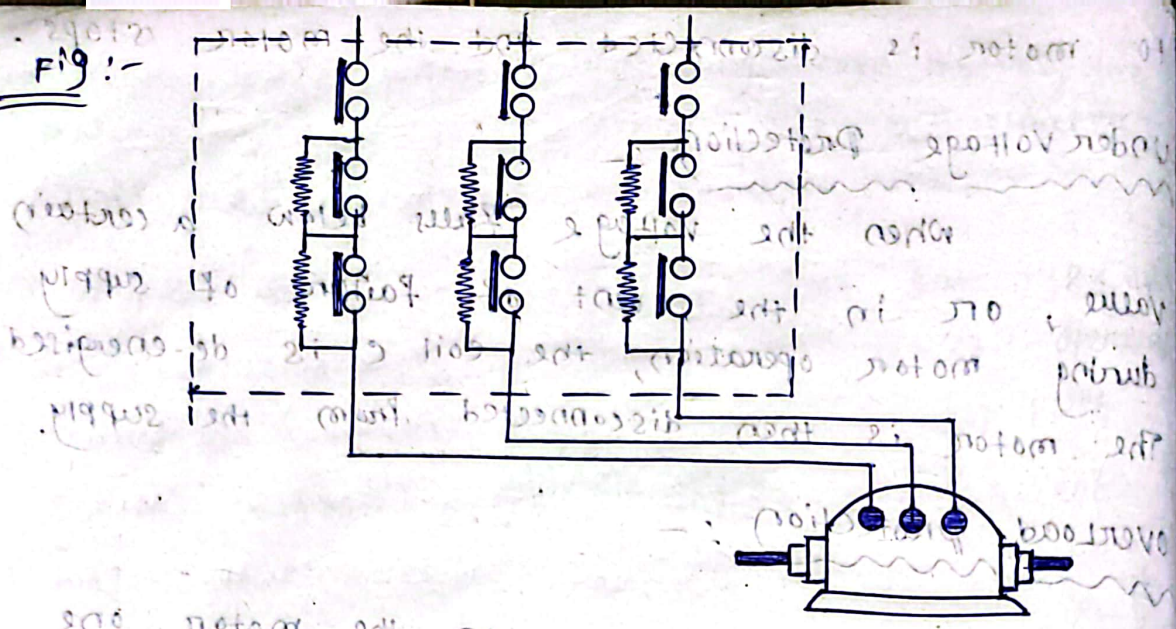
⇒ In this method, external resistances are connected in series with each phase of stator winding during starting.

⇒ This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current.

⇒ The starting resistances are gradually cut out in steps (2 or more steps) from the stator ckt as the motor picks up speed.

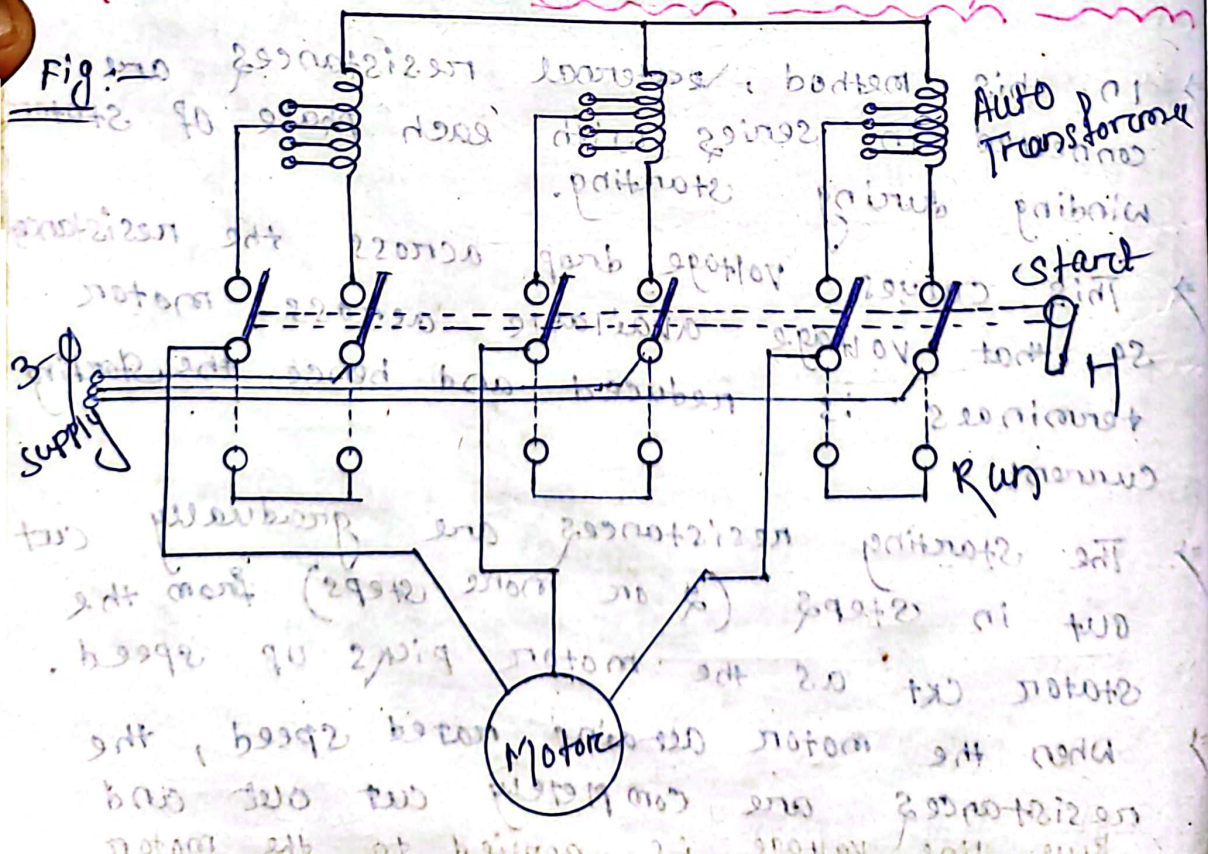
⇒ When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the motor.

Fig 1:-



- ⇒ This method suffers from 2 drawbacks, 1st -
 The reduced voltage applied to the motor during the starting period lowers the starting torque.
 ⇒ 2ndly, a lot of power is wasted in the starting resistances.

(2) Auto T/F starting :-



⇒ An AUTO T/F starter is suitable for both star & delta connected motors. In this method, the starting current is limited by using a 3- ϕ Auto T/F to reduce the initial stator applied voltage.

⇒ Fig 8 shows the motor with the auto T/F starter. The auto T/F is provided with a no. of tappings.

⇒ The starter is connected to one particular tapping to obtain the most suitable starting voltage.

A double throw switch S is used to connect the auto-T/F in the CRT for starting.

⇒ When the handle H of the switch S is in the START position, the primary of the auto-T/F is connected to the supply line and the motor is connected to the secondary of the auto-T/F.

⇒ When the motor picks up the speed, about 80% of its rated value, the handle H is quickly moved to the RUN position.

⇒ The auto T/F is disconnected from the circuit and the motor is directly connected to the line & gets its full rated voltage.

⇒ The handle is held in the RUN position by the under-voltage relay.

⇒ In case the supply voltage falls or falls below a certain value, the handle is released and returns to the OFF position.

For large (over 25 h.p), this method m/c of starting is used.

(4) STAR-DELTA STARTER (γ - Δ) :-

$\Rightarrow$ A (γ - Δ) starter is used for a cage motor designed to run normally on Δ -connected stator winding. Fig shows the connection of a 3- ϕ I.M with a γ - Δ starter.

$\Rightarrow$ When the switch S is in the START position, the stator windings are connected in STAR. When the motor picks up speed, say 80% of its rated value, the changeover switch S is thrown quickly to the RUN position which connects the stator winding in DELTA.

γ	Δ
$V_L = \frac{V_{ph}}{\sqrt{3}}$	$V_L = V_{ph}$
$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$

$\Rightarrow$ By connecting the stator windings, 1st in star and then in delta, the line current drawn by the motor at starting is reduced in ~~star~~ then in delta, the line current drawn by the motor at starting is reduced to one-third ^{or to} compared to starting current with the windings connected in delta. (25 hp)

$\Rightarrow$ At the time of starting when the stator windings are γ connected, each stator phase gets a voltage $V_L/\sqrt{3}$, where V_L = Line voltage.

method of slip ring motor starter :-

(5) Rotor Resistance starter

$\Rightarrow$ The slip-ring motors are invariably started by rotor resistance starting.

$\Rightarrow$ In this method, a variable γ -connected rheostat is connected in the rotor ckt

through slip rings and full voltage is applied to the stator winding as shown in fig.

⇒ At starting, the handle of rheostat is set in the OFF position so that max^m resistance is placed in each phase of the rotor ckt. This reduces the starting current and at the same time starting torque is increased.

⇒ As the motor picks up speed, the handle of rheostat is gradually moved in clockwise dirⁿ and cuts out the external resistance in each phase of the rotor ckt. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistor is cut out from the rotor ckt.

(fig of Y-Δ & Rotor Resistance starter):-

Speed Control of Induction motor

The speed of an i.m can be changed under 2 main headings.

- (i) control from stator side
- (ii) control from rotor side

(i) control from stator side:-

- (a) By changing the applied voltage
- (b) By changing the applied frequency (F)
- (c) By changing the no. of stator poles (P)

(ii) control from rotor side:-

- (a) Rotor Rheostatic control
- (b) Cascade operation
- (c) By injecting emf in the rotor ckt.

The relation between motor speed (N_r), syn. speed (N_s) & slip (s) is given by :-

$$N_r = (1-s) N_s$$

$$\Rightarrow N_r = (1-s) \frac{120F}{P}$$

(a) By changing applied voltage :-

Torque developed (T) by an i.m. is directly proportional to the square of applied voltage (V) i.e. $T \propto V^2$. Therefore, by changing the applied voltage, the torque and hence speed (slip) of the motor can be changed.

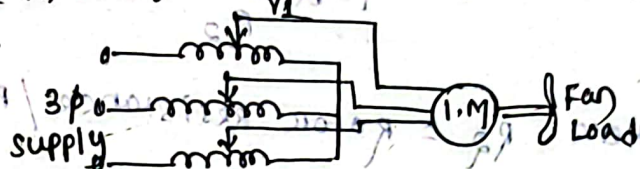
The motor is supplied from 3- ϕ supply through a 3- ϕ auto-transformer. The motor drives fan load. The voltage applied to the motor can be changed by the

autotransformer and hence desired motor speed can be obtained.

$\Rightarrow$ Let $V = 400V$, and the motor is running at speed N_1 . If the applied voltage is reduced to $0.9V$, the torque will be reduced from T to $0.81T$ ($\because T \propto V^2$). Since the torque is reduced, the motor cannot continue to run at speed N_1 . The motor speed will decrease (i.e. slip will increase).

Limitation:- Easy & cheapest method.

- A large change in voltage is required for a relatively small change in speed.
- The large change in voltage results in large change in the flux density.



(b) By changing the applied frequency (f):-

$\Rightarrow$ The synchronous speed of an 1.5 MW is given by

$$N_s = 120f/p.$$

Therefore, by changing the line

frequency f , the syn. speed (N_s) of the motor

and hence the running speed (N) can be changed.

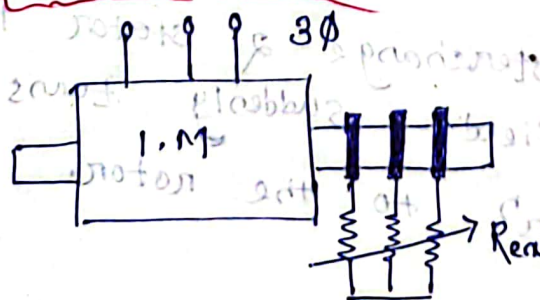
(c) By changing the no. of stator poles (p):-

$\Rightarrow$ The syn. speed of 1.5 MW $N_s = 120f/p$. Therefore by changing the no. of stator poles (p), the syn. speed and hence the motor speed (N) can be

changed. This method is easily

Control from rotor side:-

(a) rotor Rheostatic control:-



⇒ This method of speed control is suitable only for slip-ring motors. The speed of the motor can be decreased by adding external resistance to the rotor as shown in (Fig. 1.10) T.M.S.T.

⇒ Under normal running condition, the relation between torque (T) and slip (S) of an induction motor is given by

$$T \propto \frac{S}{R_2} \quad \text{or} \quad T \propto \frac{R_2}{S}$$

Here R_2 = Rotor Resistance / phase.

It is clear from the above relation that for a given torque, $S \propto R_2$. Therefore slip can be increased (i.e. motor speed can be decreased) by increasing the rotor resistance.

④ ~~Cascade operation~~

- Plugging of an I.M. :-

⇒ In some industrial applications, it is desired to bring the running induction motor to a rapid stop.

⇒ This can be done by simply interchanging the 2 stator leads. This process is called plugging.

⇒ When we interchange 2 stator leads, the revolving field suddenly turns in the opposite direction to the rotor.

Induction Generator $\Rightarrow$

$\Rightarrow$ If an I.M whose stator windings are connected to a 3- ϕ line is driven by a prime-mover at a speed higher than syn. speed, it acts as a generator.

$\Rightarrow$ It converts the mechanical energy it receives from the prime-mover into electrical energy and this electrical energy is supplied to the mains. Such a m/c is called an induction generator or asynchronous generator.

$\Rightarrow$ When speed of the I.M exceeds the syn. speed, the slip becomes negative. Therefore, the relative motion betⁿ the rotor cond^s and the rotating flux is reversed and as a result, the dirⁿ of rotor e.m.f and rotor currents will also be reversed.

$\Rightarrow$ Fig. shows an I.G connected to a 3- ϕ line. The petrol engine is the prime-mover. As soon as the engine speed exceeds the syn. speed, the motor becomes a generator, delivering active power P (KW) to the electrical systems (i.e. 3- ϕ line in this case) to which it is connected.

$\Rightarrow$ However, to create its magnetic field, the motor has to absorb reactive power Q (KVAR). This power can only come from the supply lines. The reactive power Q flows in the opposite direction to

the active power p .

⇒ The active power P (kW) delivered is directly proportional to the slip above syn. speed. Thus, a higher engine speed produces a greater o/p.

⇒ An i/g will deliver power only if it is supplied with proper reactive power to create its mag. field. For this reason, an i/g is generally connected to a 3- ϕ line.

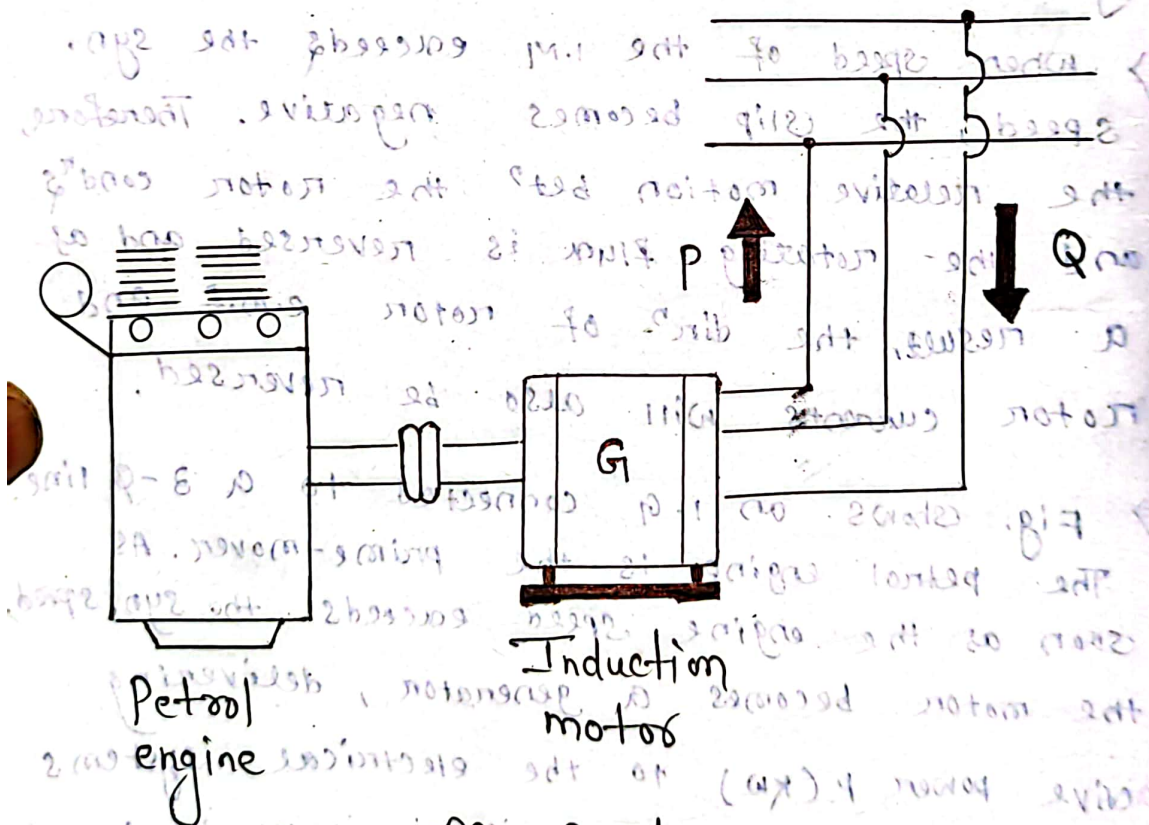


Fig: 9.3.1

Alternator

⇒ A.C. generators are usually called Alternators. They are also called synchronous generators.

⇒ A syn. generator is a m/c for converting mechanical power from a prime mover to a.c. electric power at a specific voltage & frequency. A syn. m/c rotates at a constant speed called syn. speed.

Construction of Alternator :-

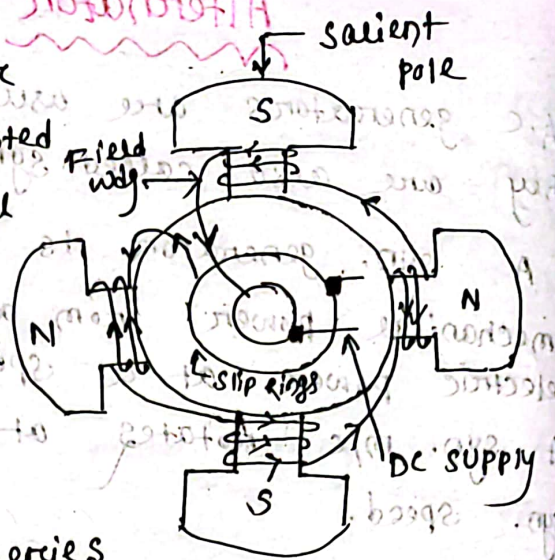
An alternator has 3- ϕ winding on the stator and d.c. field winding on the rotor.

1. Stator :- It is the stationary part of the m/c and is built up of sheet-steel laminations having slots on its inner periphery. A 3- ϕ wdg is placed in these slots and serves as the armature winding of the alternator. The arm. winding is always connected in star and the neutral is connected to ground.

2. Rotor :- The rotor carries a field wdg which is supplied with direct current through 2 slip rings by a separate d.c. source. This d.c. source (called exciter) is generally a small d.c. shunt or compound generator mounted on the shaft of the alternator. Rotor construction is of 2 types,
(a) Salient (or projecting) pole type.
(b) Non-salient (or cylindrical) pole type.

(a) Salient pole Type :-

⇒ In this type, salient or projecting poles are mounted on a large circular steel frame which is fixed to the shaft of the alternator as shown in Fig.



⇒ The individual field pole wdg are connected in series in such a way that when the field winding is energised by the d.c. exciter, adjacent poles have opposite polarities.

⇒ Low and medium speed alternators (120-400 rpm) such as those driven by diesel engines or water turbines have salient pole type rotors due to the following reasons :-

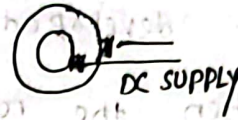
(i) The salient field poles would cause an excessive windage loss if driven at high speed and would tend to produce noise.

(ii) Salient-pole construction can not be made strong enough to withstand the mechanical stresses to which they may be subjected at higher speeds.

⇒ Since a frequency of 50 Hz is required, we must use a large no. of poles on the rotor of slow-speed alternators. Salient pole type rotor have large diameters and short axial lengths.

(b) non-salient pole type :-

⇒ In this type, the rotor is made of smooth solid forged-steel radial cylinder having a no. of slots along the outer periphery.



⇒ The field wdg are embedded in these slots and are connected in series to the slip rings through which they are energised by the d.c. exciter.

⇒ It is clear that the poles formed are non-salient i.e. they do not project out from the rotor surface.

⇒ High speed alternators (1500 or 3000 rpm) are driven by steam turbines and use non-salient type rotors due to the following reasons:-

- (i) This type of construction has mechanical robustness and gives noiseless operation at high speeds.
- (ii) The flux distribution around the periphery is nearly a sine wave and hence a better emf wave form is obtained than in the case of salient-pole type.

⇒ Since steam turbines run at high speed and a freq. of 50 Hz is required, we need a small no. of poles on the rotor of high-speed alternators (also called turboalternators)

⇒ Non-salient pole rotor have small diameters & long axial length.

Alternator Working Principle :-

⇒ The rotor winding is energised from the d.c. exciter and alternate N & S poles are developed on the rotor.

⇒ When the rotor is rotated in anticlockwise direction by a prime mover, the stator or armature conductors are cut by the magnetic flux of rotor poles.

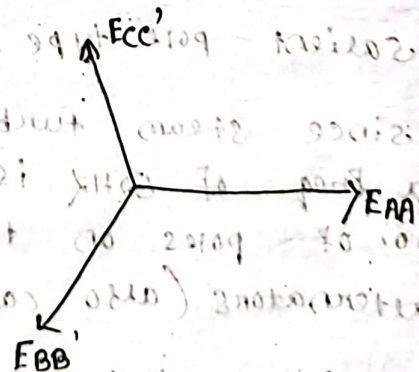
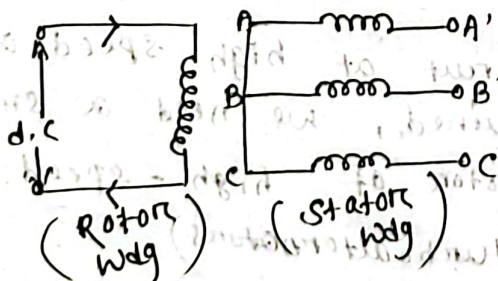
⇒ Consequently, emf is induced in the armature conductors due to electromagnetic induction. The induced emf is alternating since N and S poles of rotor alternately pass the armature cond's.

The direction of induced emf can be found by Fleming's right hand rule & freq. is given by

$$P = \frac{NP}{120}$$

Rotor - DC excitation
 excite field
 later, mech. i/p i.e. Prime mover

⇒ The magnitude of the voltage induced in each phase depends upon the rotor flux, the no. & position of the conductors in the phase and the speed of the rotor.



⇒ Fig (1) shows γ -connected arm. wdg and d.c. field wdg.

⇒ When the rotor rotated, a 3- ϕ voltage is induced in the arm. wdg. The magnitude of induced emf depends upon the speed of rotation and the d.c. exciting current. The magnitude of emf in each phase of the arm. winding is the same. However they differ in phase by 120° electrical as shown in the phasor diagram in Fig (2).

Speed & Frequency :-

The frequency of induced emf in the armature conductors depends upon speed and the no. of poles.

Let N = Rotor speed in r.p.m

p = Number of rotor poles

f = frequency of emf in Hz.

⇒ consider a stator conductor i.e. successively swept by the N & S poles of the rotor. If a +ve voltage is induced when a N-pole sweeps across the conductor, a similar -ve voltage is induced when a S-pole ~~cross~~ sweeps by. This means that one complete cycle of emf is generated in the conductor as a pair of poles passes it i.e. one N-pole and the adjacent following S-pole.

∴ No. of cycles / revolution = No. of pairs of poles

No. of revolutions / second = $N/60$

∴ No. of cycles / second = $(p/2) \left(\frac{N}{60} \right) = \frac{NP}{120}$

But no. of cycles of emf per second is

its frequency, $f = \frac{NP}{120}$

Armature winding of Alternator :-

⇒ The windings for an alternator are much simpler than that of a d.c. m/c because no commutator is used.

⇒ Fig (1) shows a 2-pole, 3- ϕ , double-layer, full-pitch, distributed winding for the stator of an alternator.

⇒ There are 12 slots and each slot contains 2 coil sides. The coil sides that are placed in adjacent slots belong to the same phase such as a_1, a_3 or a_2, a_4 constitute a phase belt.

Phase Belt :- A group of adjacent slots belonging to one phase under one pole pair is known as phase belt. The angle subtended by a phase belt is known as phase spread. The 3- ϕ wdg are always designed for 60° phase spread.

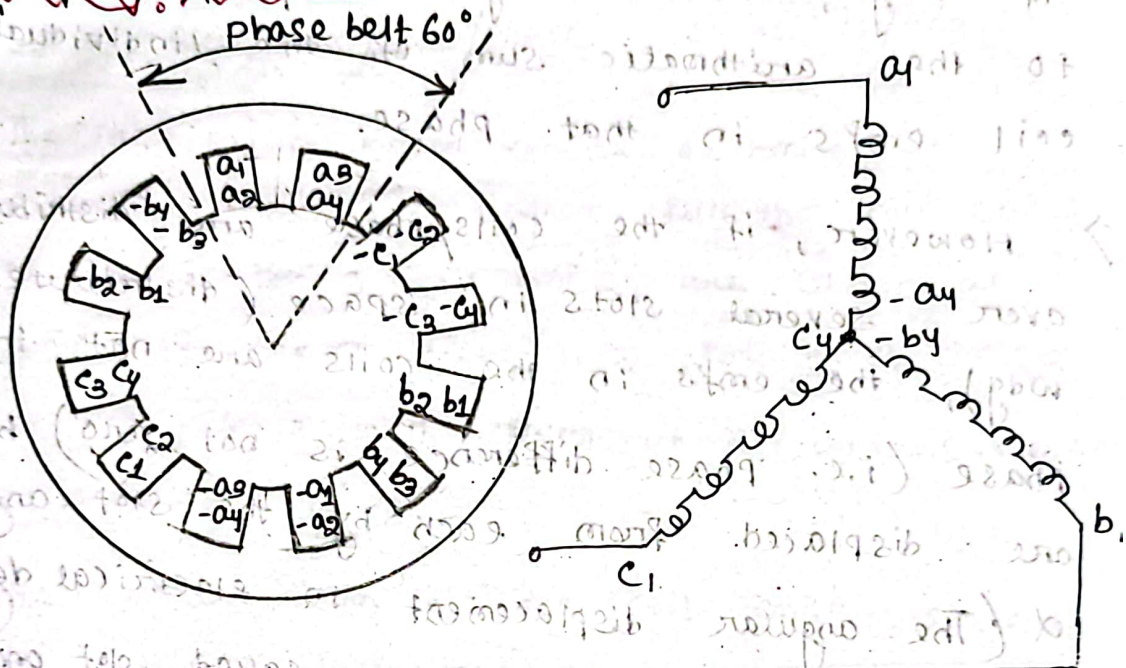
⇒ In a 3- ϕ m/c, phase belt is always 60° electrical. Since the wdg has double-layer arrangement, one side of a coil, such as a_1 is placed at the bottom of a slot and the other side a_1 is placed at the top of another slot spaced one pole pitch apart.

⇒ There are each coil has a span of a full pole pitch or 180° electrical. Therefore, the wdg is a full-pitch wdg.

⇒ There are 12 total coils and each phase has 4 coils. The 4 coils in each phase are

connected in series so that their voltages add. The 3 phases then may be connected to form a Δ connection.

Winding Factors :-



Winding Factors :-

The armature winding of an alternator is distributed over the entire armature. The distributed winding produces nearly a sine waveform and the heating is more uniform. The distribution and pitching of the coils affect the voltages induced in the coils.

- (1) Distribution Factor (K_d) / Breadth Factor
- (2) Pitch Factor (K_p) / Chord Factor.

(1) Distribution Factor (K_d) :-

$\Rightarrow$ A winding with only one slot/pole/phase is called a concentrated winding. In this type of wdg, the emf generated/phase is equal to the arithmetic sum of the individual coil emfs in that phase.

$\Rightarrow$ However, if the coils/phase are distributed over several slots in space (distributed wdg), the emfs in the coils are not in phase (i.e. phase difference is not zero) but are displaced from each by the slot angle α (The angular displacement in electrical degrees betⁿ the adjacent slots is called slot angle). The emf/phase will be the phasor sum of coil emfs.

The Distribution Factor K_d is defined as:

$$K_d = \frac{\text{emf with distributed wdg}}{\text{emf with concentrated wdg}} = \frac{\text{phasor sum of coil emfs/phase}}{\text{arithmetic sum of coil emfs/phase}}$$

numerator is less than denominator $\therefore K_d < 1$.

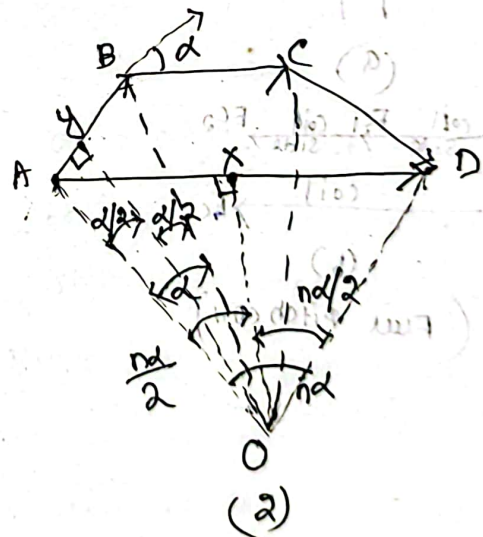
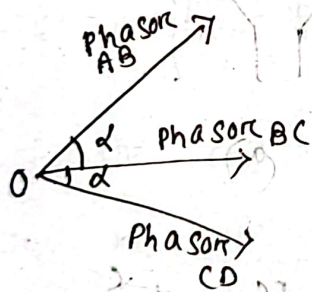
Expression for K_d :-

Let $\alpha = \text{slot angle} = \frac{180^\circ \text{ elect.}}{\text{No. of slots/pole}}$

$n = \text{slots/pole/phase}$ or $16/4/3 = 4/3$

$\Rightarrow$ The distribution factor can be determined by constructing a phasor diagram for the coil emfs. Let $n = 3$. The 3 coil emfs are shown as phasors AB, BC & CD (Fig 1) each of which is a chord of circle with centre at O, and subtends an angle α at O.

$\Rightarrow$ The phasor sum of the coil emfs subtends an angle $n\alpha$ (Here $n=3$) at O. Draw the bisectors of each chord such as ox, oy etc (Fig 2).



$$K_d = \frac{AD}{n \times AB} = \frac{2AY}{n \times (2AY)} = \frac{AY}{n \times AY}$$

$$= \frac{\phi A \sin(n\alpha/2)}{n \times \phi A \sin(\alpha/2)}$$

$n\alpha = \text{phase spread}$

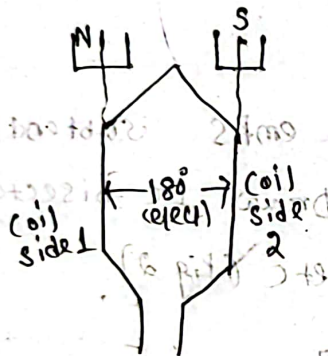
$$K_d = \frac{\sin(n\alpha/2)}{n \sin(\alpha/2)}$$

Pitch Factor (K_p) :-

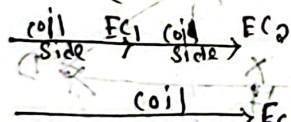
→ The distance betⁿ the 2 sides of a coil is called the coil span or coil pitch.

⇒ The angular distance betⁿ the central line of one pole to the central line of the next pole is called pole pitch. A pole pitch is always 180° electrical.

→ A coil having a span equal to 180° electrical is called a full-pitch coil (a).

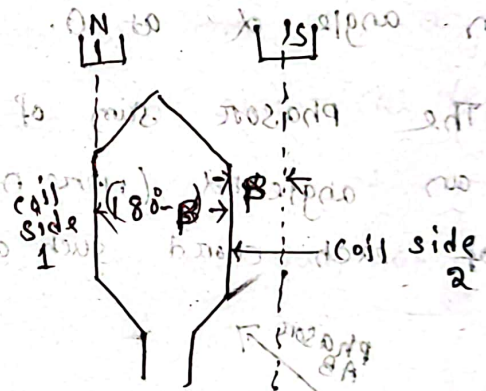


(a)

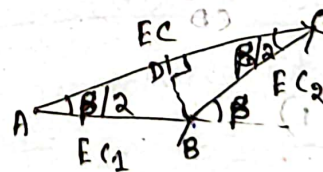


(b)

(Full pitch coil)



(a)



(b)

(Short-pitch coil)

→ A coil having a span less than 180° elect. is called a short-pitch coil, or a fractional-pitch coil. It is also called a chorded coil. A stator wdg using fractional-pitch coils is called a chorded wdg. If the span of the coil is reduced by an angle β electrical degrees, the coil span will be $(180 - \beta)^\circ$ elect.

⇒ In case of a full-pitch coil, the 2 coil sides span a distance exactly equal to the pole pitch of 180° elect. As a result, the voltage generated in a full-pitch coil is such that the coil-side voltages are in phase as shown in Fig(1). Let E_{c1} & E_{c2} = voltages generated in the coil sides & E_c = resultant coil voltage. Then:

$$E_c = E_{c1} + E_{c2}$$

$$|E_{c1}| = |E_{c2}| = E_1 \text{ (say)}$$

Since E_{c1} & E_{c2} are in phase, the resultant coil voltage E_c is equal to their arithmetic sum.

$$E_c = E_{c1} + E_{c2} = 2E_1$$

⇒ If the coil span of a single coil is less than the pole pitch of 180° elect, the voltages generated in each coil side are not in phase. The resultant coil voltage E_c = phasor sum of E_{c1} & E_{c2} .

If the coil span is reduced by an angle β° elect, the coil span is $(180 - \beta)^\circ$ elect. The voltages generated E_{c1} & E_{c2} in the 2 coil sides will be out of phase with each other by an angle β° elect as shown in Fig(2). The phasor sum of E_{c1} & E_{c2} is $E_c (= AC)$.

⇒ The coil span - Factor or pitch - Factor k_c is defined as the ratio of the voltage generated in the short - pitch coil to the voltage generated in the full - pitch coil. The coil span - Factor is also called the chording factor.

$$k_c = \frac{\text{Actual voltage generated in the coil}}{\text{voltage generated in the coil of span } 180^\circ \text{ elec.}}$$

$$= \frac{\text{emf induced in short-pitch coil}}{\text{emf induced in full-pitch coil}}$$

$$= \frac{\text{phasor sum of the emf of 2 coil side}}{\text{arithmetic sum of the emf of 2 coil sides}}$$

$$= \frac{\frac{AC}{2AB}}{\frac{2AD}{2AB}} = \frac{2 \times AB \cos \beta/2}{2AB}$$

$$k_c = \cos \beta/2$$

For full - pitch coil, $\beta = 0^\circ$, $\cos \frac{\beta}{2} = 1$

$k_c = 1$ For short pitch coil $k_c < 1$

EMF Equation of an Alternator:-

Let Z = No. of conductors on coil sides in series / phase.

ϕ = Flux/pole in wbs.

P = No. of rotor poles.

N = Rotor speed in r.p.m.

(in one revolution, i.e. $\frac{60}{N}$ sec.), each stator condⁿ is cut by $P\phi$ wbs i.e.

$$d\phi = P\phi; dt = \frac{60}{N}$$

$\therefore$ Avg. emf induced in one stator condⁿ

$$= \frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{P\phi N}{60} \text{ Volts}$$

Since there are Z condⁿs in series/phase.

$$\text{Avg. emf/phase} = \frac{P\phi N}{60} \times Z$$

$$= \frac{P\phi Z}{60} \times \frac{120P}{P} \quad (\because N = \frac{120P}{P})$$

$$= 2P\phi Z \text{ Volts.}$$

R.M.S value of emf/phase = Avg. value/phase $\times$ Form Factor

$$= 2P\phi Z \times 1.11$$

$$= 2.22P\phi Z \text{ Volts}$$

$$\therefore E_{rms}/\text{phase} = 2.22P\phi Z \text{ Volts} \quad \text{--- (1)}$$

If K_p & K_d are the pitch factor & distribution factor of the arm. wdg, then

$$E_{rms}/\text{phase} = 2.22K_p K_d P\phi Z \text{ Volts} \quad \text{--- (2)}$$

Sometimes the turns (T) / phase rather than conductors / phase are specified. In that case, eq. (2) becomes

$$E_{rms} / \text{phase} = 4.44 K_p K_d F \phi T \text{ volts.} \quad \text{--- B)}$$

The line voltage will depend upon whether the winding is y or Δ connected.

EFFECT OF HARMONICS ON PITCH & DISTRIBUTION FACTOR

(a) IF the short-pitch angle or chording angle is

β° for the fundamental flux wave, then its values for different harmonics are

$$\text{For 3rd harmonic} = 3\beta$$

$$\text{" 5th " } = 5\beta$$

$$\therefore \text{pitch factor, } K_p = \cos \beta/2 \quad (\text{For fundamental})$$

$$= \cos 3\beta/2 \quad (\text{For 3rd harmonic})$$

$$= \cos 5\beta/2 \quad (\text{For 5th harmonic})$$

(b) Similarly, the Distribution Factor is also

different for different harmonics

$$K_d = \frac{\sin\left(\frac{n\alpha}{2}\right)}{n \sin\left(\frac{\alpha}{2}\right)}$$

where m = order of the harmonic

$$\text{For fundamental } n=1 \quad K_{d1} = \frac{\sin n\alpha/2}{n \sin \alpha/2}$$

$$\text{" 3rd harmonic, } n=3 \quad K_{d3} = \frac{\sin 3n\alpha/2}{n \sin 3\alpha/2}$$

$$\text{" 5th " } n=5 \quad K_{d5} = \frac{\sin 5n\alpha/2}{n \sin 5\alpha/2}$$

(c) Frequency is also changed. If fundamental frequency

is 50 Hz , i.e. $F_1 = 50\text{ Hz}$,

3rd harmonic, $F_3 = 3 \times 50 = 150\text{ Hz}$

5th " $F_5 = 5 \times 50 = 250\text{ Hz}$ etc.

Harmonics :-

The presence of harmonics in electrical systems means that current and voltage are distorted and deviate from sinusoidal waveforms. Harmonic currents are caused by non-linear loads connected to the distribution system.

How ~~Does~~ the Harmonics are generated :-

Harmonics are multiples of the fundamental wavelength. Thus, the 3rd order harmonic is the third multiple of the fundamental wavelength. This type of harmonics is generated in non-linear loads, i.e. motors, transistors and the non-ideal T/F S.

What are the causes of harmonics :-

Harmonics are caused by non-linear loads, i.e. loads that draw a non-sinusoidal current from a sinusoidal voltage source. Some examples of harmonic producing loads are inverters, DC converters, smps, AC/DC motor drive.

Armature Reaction in Alternator :-

⇒ The effect of armature flux on the flux produced by field ampere-turns (i.e. rotor ampere-turns) is called armature reaction.

⇒ To illustrate this consider the following 3 cases :- (a) when load p.f. is unity.

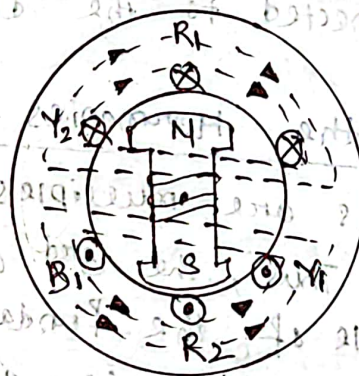
(b) when load p.f. is zero lagging.

(c) when load p.f. is zero leading.

(a) when load p.f. is unity



(1)



⇒ Fig (1) shows an elementary alternator on no-load. Since the armature is on open-ckt, there is no stator current and the flux due to rotor current is distributed symmetrically in the air-gap as shown in Fig (1).

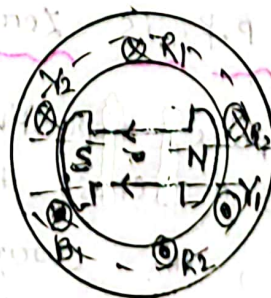
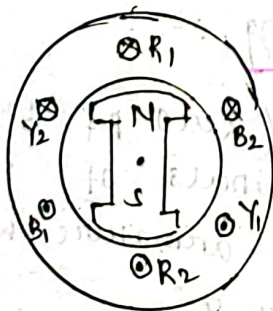
⇒ Since the direction of the rotor is assumed clockwise, the generated emf in phase $R_1 R_2$ is at its max^m and is towards in the condⁿ R_1 and outwards in condⁿ R_2 . No arm. flux is produced since no current flows in the arm. wdg.

Fig (2) shows the effect when a resistive load (unity p.f.) is connected across the terminals of the alternator. According to Right-hand Rule, the current is "in" in the conductors under N-pole and "out" in the conductors under S-pole. Therefore, the arm. flux is clockwise due to currents in the top conductors and anticlockwise due to currents in the bottom conductors. Note that arm. flux is at 90° to the main flux (due to rotor current) and is behind the main flux.

$\Rightarrow$ In this case, the flux in the air-gap is distorted but not weakened. Therefore, at unity p.f., the effect of arm. reaction is merely to distort the main field, there is no weakening of the main field and the avg flux practically remains the same.

Since the magnetic flux due to stator currents (i.e. arm. flux) rotates synchronously with the rotor, the flux distortion remains the same for all positions of the rotor.

(b) When Load p.f. is zero Lagging :-



⇒ When a pure inductive load (Zero p.f. lagging) is connected across the terminals of the alternator, current lags behind the voltage by 90° .

⇒ Fig (i) shows the condition when the alternator is supplying resistive load. The emf as well as current in phase $R_1 R_2$ is ϕ_{max} in the position shown. When the alternator is supplying a pure inductive load, the current in phase $R_1 R_2$ will not reach its ϕ_{max} value until N-pole advances 90° electrical as shown in Fig (2). Now the arm. flux is from right to left & field flux is from left to right.

⇒ All the flux produced by arm. current (i.e. arm. flux) opposes the field flux and

therefore, weakens it. In other words, arm. reaction is directly demagnetising.

⇒ Hence at zero p.f. lagging, the arm. reaction weakens the main flux. This causes a reduction in the generated emf.

(c) When load p.f. is Zero Leading :-

⇒ When a pure capacitive load (Zero p.f. Leading) is connected across the terminals of the alternator, the current in armature wdg will lead the induced emf by 90° .

⇒ Obviously, the effect of armature reaction will be the reverse that for pure inductive load. Thus, armature flux now aids the main flux and the generated emf is increased.

⇒ Fig (1) shows the condⁿ when alternator is supplying resistive load, emf as well as current in phase R_1, R_2 is max^m in the position.

When the alternator is supplying a pure capacitive load, the max^m current in R_1, R_2 will occur 90° electrical before the occurrence of max^m induced emf. Therefore, max^m current

in phase R_1, R_2 will occur if the posⁿ of the rotor remains 90° behind as compared to its position under resistive load.

⇒ It is clear that arm. flux is now in the same direction as the field flux and

therefore, strengthens it. This causes an increase in the generated voltage. Hence

at zero p.f. leading the arm. reaction strengthens the main flux.

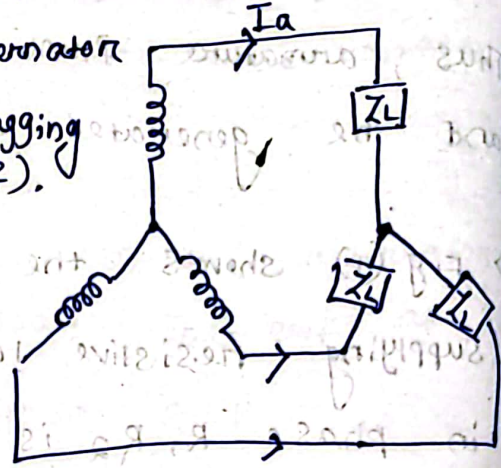
Alternator on Load :-

→ Fig shows γ -connected alternator supplying inductive load (Lagging p.f.).

When the load on the alternator is increased

(i.e. arm. current I_a is increased), the field

excitation and speed being kept constant, the terminal voltage V (phase value) of the alternator decreases. This is due to



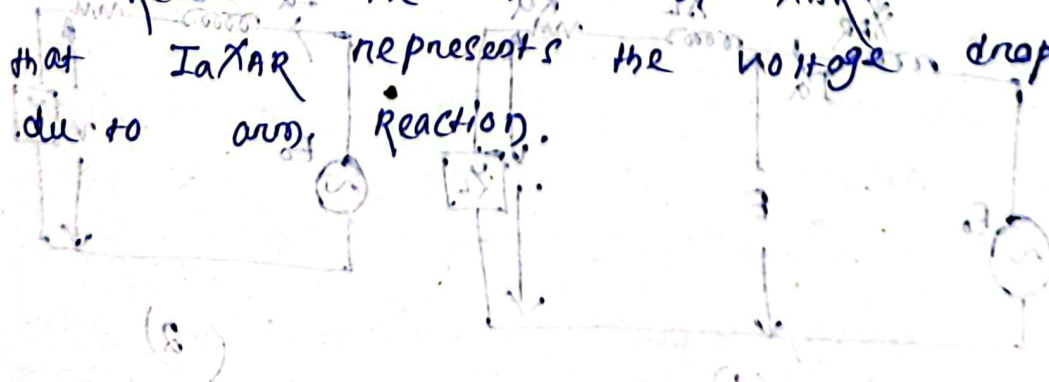
- (i) voltage drop $I_a R_a$ where $R_a =$ arm. Resistance / phase.
- (ii) voltage drop $I_a X_L$ where $X_L =$ arm. Leakage Reactance / phase.
- (iii) voltage drop because of arm. Reaction.

(i) Arm. Resistance (R_a) :- Since the arm. or stator wdg has some resistance, there will be an $I_a R_a$ drop when current (I_a) flows through it. The arm. resistance / phase is generally small so that $I_a R_a$ drop is negligible for all practical purposes.

(ii) Arm. Leakage Reactance (X_L) :- when current flows through the arm. wdg,

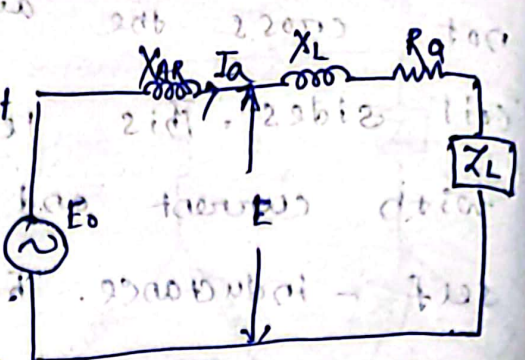
flux is set up and a part of it does not cross the air-gap and links the coil sides. This leakage flux alternates with current and gives the winding self-inductance. This is called arm. Leakage reactance. Therefore, there will be $I_a X_L$ drop which is also effective in reducing the terminal voltage.

(35) Armature Reaction :- The load is generally inductive and the effect of arm. reaction is to reduce the generated voltage. Since arm. reaction results in a voltage effect in a ckt caused by the change in flux produced by current in the same ckt, its effect is of the nature of an inductive reactance. Therefore, arm. reaction effect is accounted for by assuming the presence of a fictitious reactance X_{AR} in the arm. ckt. The quantity X_{AR} is called reactance of arm. reaction. The value of X_{AR} is such that $I_a X_{AR}$ represents the voltage drop due to arm. reaction.



Equivalent ckt :-

Fig shows the equivalent ckt of the loaded alternator for one phase.



All the quantities are/phase

$E_0 = \text{No-load emf}$

$E = \text{Load induced emf. It is the induced emf after allowing for}$

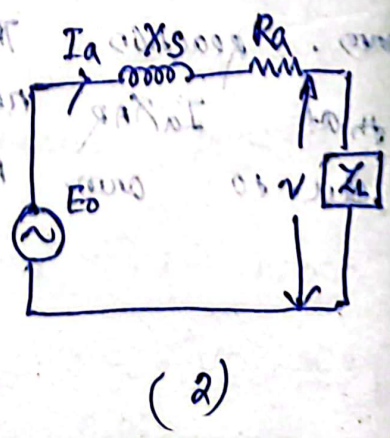
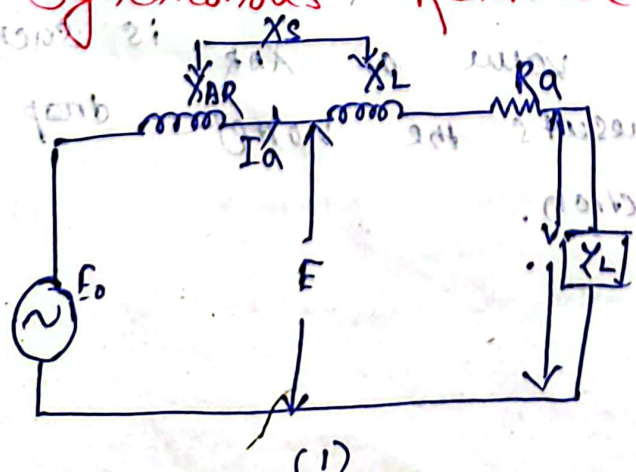
arm reaction. It is equal to phasor difference of E_0 & $I_a X_{AR}$.

$V = \text{Terminal voltage. It is less than } E \text{ by voltage drops in } X_L \text{ and } R_a.$

$E = V + I_a (R_a + jX_L)$

$E_0 = E + I_a (jX_{AR})$

Synchronous Reactance :-



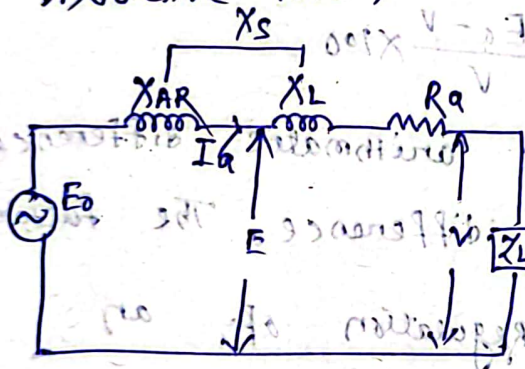
⇒ The sum of arm. Leakage reactance (X_L) & reactance of arm. reaction (X_{AR}) is called syn. reactance X_s .

syn. impedance $Z_s = R_a + jX_s$

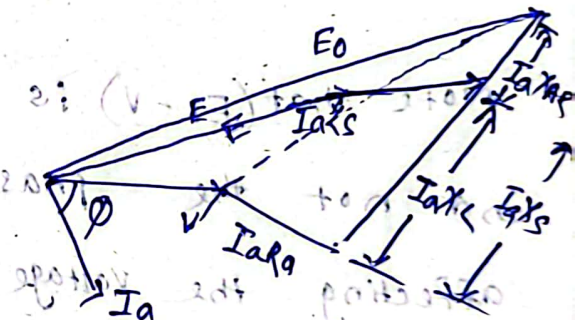
$$E_0 = (V + I_a Z_s) \\ = V + I_a (R_a + jX_s)$$

Phasor Diagram of a Loaded Alternator:

consider a γ -connected alternator supplying inductive load, the load p.f. angle being ϕ .



(1)



(2)

⇒ Fig (2) shows the phasor diagram of an alternator for the usual case of inductive load. The arm. current I_a lags the terminal voltage V by p.f. angle ϕ . The phasor sum of V and drops $I_a R_a$ & $I_a X_L$ gives the load induced voltage E . It is the induced emf after allowing for armature reaction. The phasor sum of E & $I_a X_{AR}$ gives the no-load emf E_0 .

⇒ Note:- While drawing the phasor diagram, either the terminal voltage (V) or arm. current (I_a) may be taken as reference phasor.

Voltage Regulation :-

The voltage regulation of an alternator is defined as the change in terminal voltage from no-load to full-load (the speed & field excitation being constant) divided by full-load voltage.

$$\% \text{ voltage regulation} = \frac{\text{No-load voltage} - \text{Full-load voltage}}{\text{Full-load voltage}} \times 100$$
$$= \frac{E_0 - V}{V} \times 100$$

Note that $(E_0 - V)$ is the arithmetic difference and not the phasor difference. The factors affecting the voltage regulation of an alternator are :-

(i) $I_a R_a$ drop in arm. wdg.

(ii) $I_a X_L$ drop

(iii) voltage change due to arm. reaction.

Determination of Voltage Regulation :-

Several methods are used for this.

(1) Synchronous impedance or EMF method.

(2) Ampere-turn or MMF method.

(3) Zero p.f. or Potier method.

For these methods, the following data are required:

(i) Arm. Resistance

(ii) O.C.C.

(iii) S.C.C.

(i) Arm. Resistance :- The arm. Resistance R_a /phase is determined by using direct current and the voltmeter - ammeter method. This is the d.c. value. The effective arm. resistance (a.c. resistance) is greater than this d.c. value. It is usual practice to take the effective resistance 1.5 times the d.c. value ($R_a = 1.5 R_{dc}$).

(ii) O.C.C. :- It is the curve between arm. terminal voltage (phase value) on open ckt and the field current when the alternator is running at rated speed (syn. speed).

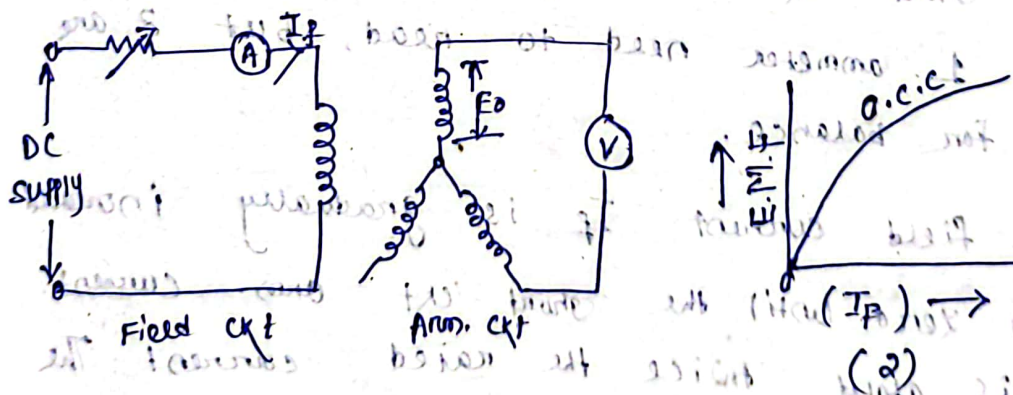
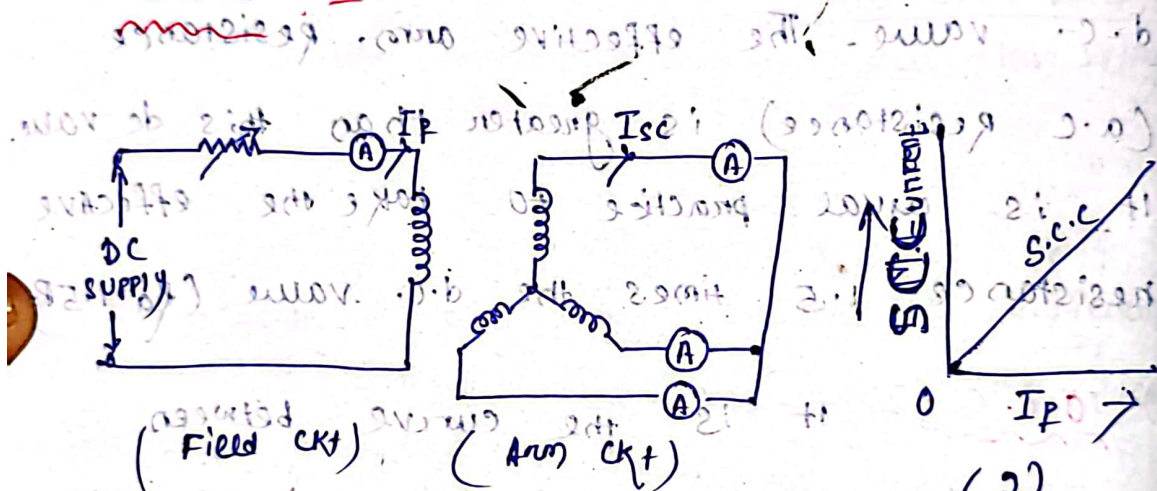


Fig (1) shows the ckt for determining the O.C.C. of an alternator.

The alternator is run on no-load at the rated speed. The field current I_f is gradually increased from zero (by adjusting field rheostat) until open-ckt voltage E_o (phase value) is about 50% greater than the rated phase voltage. (iii)

⇒ The graph is drawn betⁿ open-ckt voltage values & the corresponding values of I_f as shown in Fig (2).

(iii) S.C.C.



⇒ In a short-ckt test, the alternator is run at rated speed and the arm. terminals are short-cktd through identical ammeters. Only 1 ammeter need to read, but 3 are used for balance.

⇒ The field current I_f is gradually increased from zero until the short-ckt arm. current I_{sc} is about twice the rated current. The

graph betⁿ short-ckt arm current & field current gives the S.S.C. as shown (2).

Synchronous Impedance method :-

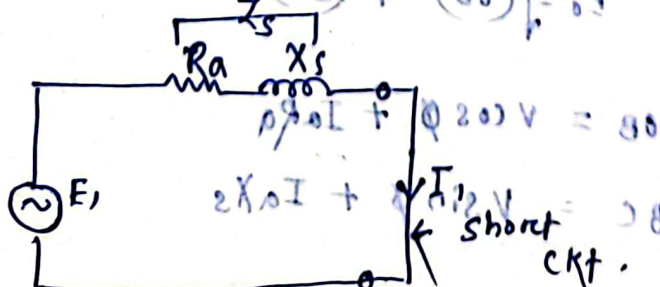
→ In this method of finding the voltage regulation of an alternator, we find the syn. impedance Z_s (and hence syn. reactance X_s) of the alternator from the o.c.c. & s.c.c. For this reason, it is called syn. impedance method. The method involves the following steps

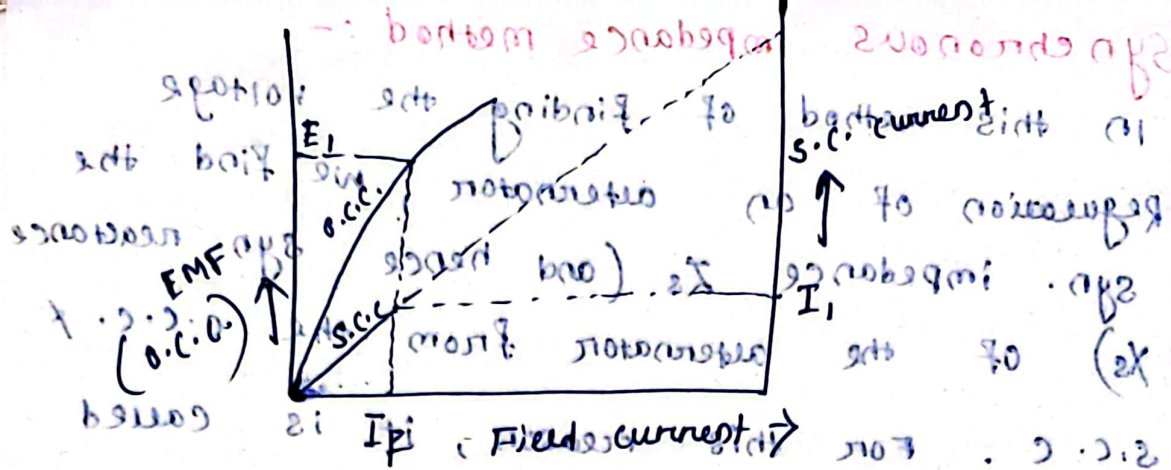
(1) Plot the o.c.c. & s.c.c. on the same field current.

(2) Consider a field current I_f . The open-ckt voltage corresponding to this field current is E_1 . The short-ckt armature current corresponding to field current I_f is I_1 on short-ckt. E_1 is being used to circulate the short-ckt arm. current I_1 against the syn. impedance Z_s .

$$\Rightarrow Z_s = \frac{E_1 \text{ (open-ckt)}}{I_1 \text{ (short-ckt)}}$$

Note that E_1 is the phase value and so is I_1 .

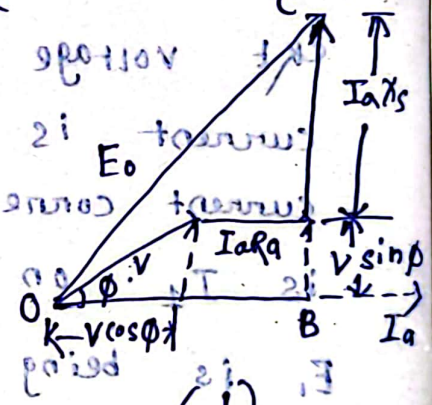




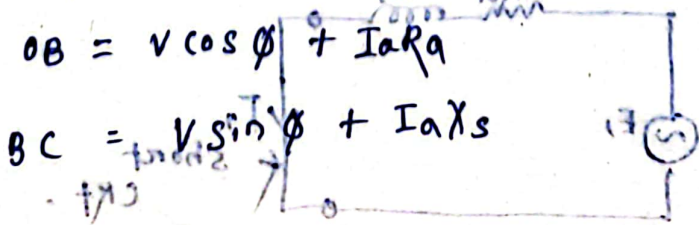
(3) The arm. resistance can be found as follows
 explained

$$\therefore \text{syn. Reactance, } X_s = \sqrt{Z_s^2 - R_a^2}$$

(4) Fig (1) shows the phasor diagram for the inductive load, the load pf being $\cos \phi$ lagging. current I_a has been taken as the reference phasor. The $I_a R_a$ drop is in phase with I_a while $I_a X_s$ drop leads I_a by 90° . The phasor sum of V , $I_a R_a$ and $I_a X_s$ gives the no-load emf E_0



$$E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$



$$E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

$$\therefore \% \text{ voltage Regulation} = \frac{E_0 - V}{V} \times 100$$

Drawback :- This method is easy but it gives approximate results. The reason is simple. The combined effect of X_L (arm. leakage reactance) and X_{AR} (reactance of arm. reaction) is measured on short-circuit.

→ This method gives a value higher than the value obtained from an actual load test. For this reason, it is called pessimistic method.