

SUBJECT-ENGINEERING PHYSICS
SEMESTER-1ST & 2ND

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UNIT-5

Gravitation

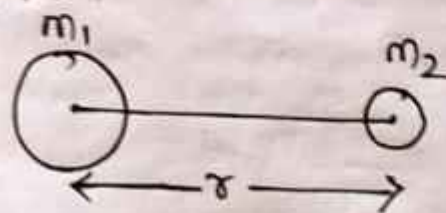
- Newton's Laws of gravitation - Statement & Explanation
- Universal Gravitational Constant (G) - Definition, Unit & Dimension
- Acceleration due to gravity (g) - Definition and concept
- Definition of mass and weight
- Relation between g and G
- Variation of g with altitude and depth (No derivation - Only Explanation)
- Kepler's Laws of planetary Motion (Statement only)

Newton's Laws of Gravitation ⇒ Statement ⇒

Every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

This law is often termed as Inverse Square Law.

Explanation ⇒



Consider two bodies of masses m_1 and m_2 . & r be the distance between their centres.

Let F be the magnitude of the force of attraction between them.

According to the Newton's law of gravitation,

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

Combining the above two factors, we get

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \boxed{F = G \frac{m_1 m_2}{r^2}} \quad \text{--- (1)}$$

Where G is a constant of proportionality which is known as Universal Gravitational Constant.

Definition of G

$$\text{From eqn- (1), } G = \frac{F r^2}{m_1 m_2}$$

$$\text{If } m_1 = m_2 = 1 \text{ unit,}$$

$$r = 1 \text{ unit,}$$

$$\text{Then, } G = \frac{F \times 1^2}{1 \times 1}$$

$$\Rightarrow \boxed{G = F}$$

The universal gravitational constant may be defined as the magnitude of force of attraction between two bodies each of unit mass and separated by a unit distance from each other.

Unit of G $\Rightarrow$

(i) CGS system:

In CGS system, the unit of G is $\text{dyne cm}^2 \text{g}^{-2}$.

(ii) S.I. system:

In S.I. system, the unit of G is $\text{Nm}^2 \text{kg}^{-2}$.

Value of G $\Rightarrow$

(i) In CGS system, $G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{g}^{-2}$

(ii) In S.I. system, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$

Dimensions of G $\Rightarrow$

$$\begin{aligned} G &= \frac{F \times r^2}{m_1 \times m_2} \\ &= \frac{[M^1 L^1 T^{-2}] \times [L^2]}{[M^1] [M^1]} \\ &= [M^{-1} L^3 T^{-2}] \end{aligned}$$

Therefore, the dimensions of G are (-1), 3 & (-2) in mass, length & time respectively.

Properties of gravitational Force $\Rightarrow$

(1) Gravitational force of attraction between any 2 bodies are equal in magnitude but opposite in direction.

They are a pair of action and reaction.

(2) Gravitational force of attraction between any two bodies is independent of the presence of other bodies in between them.

- (3) Gravitational force of attraction is independent of the nature of the medium in between the 2 bodies.
- (4) Gravitational force of attraction is very small for small bodies and is appreciable for massive bodies.
- Gravitational force exist between every objects on this earth, but they can not attract each other due to very small force of attraction.

Acceleration due to gravity (g) ⇒

$$M_s = 1.989 \times 10^{30} \text{ kg}$$

$$M_e = 5.972 \times 10^{24} \text{ kg}$$

$$R = 149.73 \times 10^9 \text{ m}$$

- The force of attraction between earth and a body near it is called gravity.
- The acceleration produced in the body due to gravity is called acceleration due to gravity.

It is denoted by ' g '.

- Value of ' g ' on the surface of the earth:

① In CGS system, $g = 980 \text{ cm/s}^2$

② In SI system, $g = 9.8 \text{ m/s}^2$

$$M_e = 5.972 \times 10^{24} \text{ kg}$$

$$R_e = 6371 \text{ km}$$

Comparison of Mass and Weight ⇒

Mass

Weight

(1) It is the amount of matter contained in a body.

(1) It is the force with which a body is attracted towards the centre of earth.

(2) It is a scalar quantity.

(2) It is a vector quantity.

(3) In S.I., it is measured in kg.

(3) In S.I., it is measured in newton.

(4) It is a constant quantity.

(4) It may vary from place to place since it depends upon the value of 'g' at a place.

(5) It is never zero for a material body.

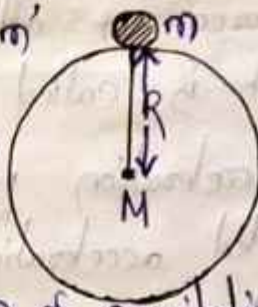
(5) It is zero at the centre of earth. It may also be zero in some

(6) Pan balance ^{or electronic balance} ~~are~~ used to measure the mass.

(6) Spring balance or weighing ^{where no gravity acts upon object} machine are used to measure the weight.

Relation between g and G

Let us consider a body of mass 'm' is placed on the surface of the earth of mass 'M' and radius 'R'.



Then, according to Newton's law of gravitation, the force of attraction (gravity) between the body & the earth is given by.

$$F = G \frac{Mm}{R^2} \quad \text{--- (1)}$$

Weight of the body is given by,

$$F = mg \quad \text{--- (2)}$$

But, the weight of body and the gravity both are similar things.

So, from eq (1) and (2),

$$mg = G \frac{Mm}{R^2}$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \quad \text{--- (3)}$$

From eqⁿ ③, it is clear that 'g' depends upon mass of planet and radius of planet, so it is not a universal constant.

Variation of g with altitude $\Rightarrow$

Let us consider a body of mass 'm' is placed on the surface of the earth having mass 'M'

Let R be the radius of the earth.

The acceleration due to gravity on the surface of earth is given by

$$g = \frac{GM}{R^2} \text{ --- ①}$$

Suppose the body is taken to a height 'h' from the surface of earth.

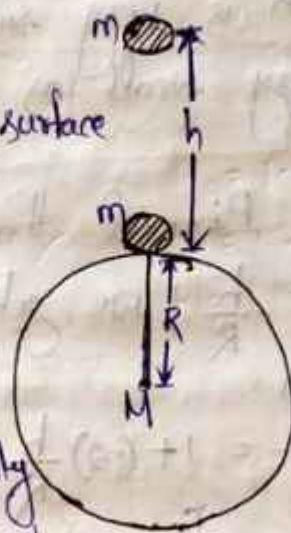
Let g' be the acceleration due to gravity at a height 'h' from the surface of earth.

$$\text{Then, } g' = \frac{GM}{(R+h)^2} \text{ --- ②}$$

Dividing equation-② by eqⁿ ①, we get

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{[R(1+\frac{h}{R})]^2}$$



$$\Rightarrow \frac{g'}{g} = \frac{R^2}{R^2 \left(1 + \frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2} \quad \text{--- (3)}$$

Since, h is very small as compared to R , therefore $\frac{h}{R}$ is very small as compared to 1.

Applying Binomial theorem and neglecting the higher powers of $\frac{h}{R}$, we get

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + x^4 + \dots + x^n$$

$$\begin{aligned} \frac{g'}{g} &= 1 + (-2)\frac{h}{R} \\ &= 1 - \frac{2h}{R} \end{aligned}$$

$$\Rightarrow \boxed{g' = g\left(1 - \frac{2h}{R}\right)} \quad \text{--- (4)}$$

The value of g on the surface of earth is constant. So when ' h ' increases, the term $\frac{2h}{R}$ increases and g' decreases. Therefore, the value of acceleration due to gravity decrease whenever one moves upward from the surface of earth.

Loss in weight at a height ' h ' $\Rightarrow$

$$\text{At a height 'h', } g' = g\left(1 - \frac{2h}{R}\right)$$

Multiplying mass of the body ' m ' on both the sides of the above eq,

$$mg' = mg\left(1 - \frac{2h}{R}\right)$$

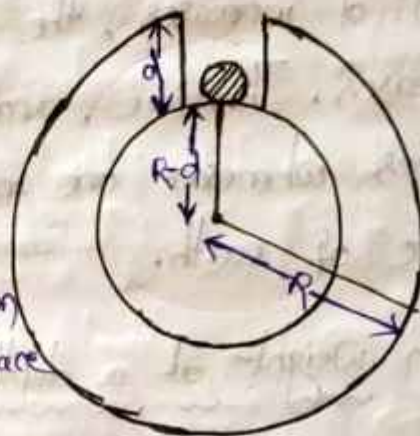
$$\Rightarrow mg' = mg - \frac{2mgh}{R}$$

$$\Rightarrow mg - mg' = \frac{2mgh}{R}$$

$$\Rightarrow \boxed{\text{Loss in weight} = \frac{2mgh}{R}}$$

Variation of g with depth $\Rightarrow$

Let us consider a body of mass ' m ' is placed on the surface of earth of mass ' M ' and radius ' R '.



Then, the value of acceleration due to gravity on the surface of earth is given by

$$g = \frac{GM}{R^2} \quad \text{--- (1)}$$

$$\Rightarrow g = \frac{G \times \frac{4}{3} \pi R^3 \rho}{R^2} \quad (\because \text{where } \rho = \text{density of earth})$$

$$\Rightarrow \boxed{g = \frac{4}{3} \pi R G \rho} \quad \text{--- (2)}$$

Suppose the body is taken to a depth ' d ' from the surface of earth.

Let g' be the acceleration due to gravity at the depth ' d ' from the surface of earth.

$$g' = \frac{4}{3} \pi (R-d) G \rho \quad \text{--- (3)}$$

Dividing eq(3) by eq(2), we get

$$\frac{g'}{g} = \frac{R-d}{R}$$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{d}{R}$$

$$\Rightarrow \boxed{g' = g\left(1 - \frac{d}{R}\right)} \text{---(4)}$$

From eq(4), it is clear that,

When 'd' increases, the term $\frac{d}{R}$ increases and g' decreases. Therefore, acceleration due to gravity decreases whenever one moves downward from the surface of earth.

Loss in weight at a depth 'd' $\Rightarrow$

$$\text{At a depth 'd', } g' = g\left(1 - \frac{d}{R}\right)$$

Multiplying mass of the body 'm' on both the sides of the above eq.

$$mg' = mg\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow mg' = mg - \frac{mgd}{R}$$

$$\Rightarrow mg - mg' = \frac{mgd}{R}$$

$$\Rightarrow \boxed{\text{Loss in weight} = \frac{mgd}{R}}$$

Weight of a body at the center of earth $\Rightarrow$

At the center of earth, $d=R$

Putting $d=R$ in equation (4), we get

$$g' = g\left(1 - \frac{d}{R}\right)$$

$$= g\left(1 - \frac{R}{R}\right)$$

$$= g(1-1)$$

$$= g \times 0$$

$$\Rightarrow \boxed{g' = 0}$$

Therefore, the value of acceleration due to gravity is '0' at the center of earth.

If 'm' is the mass of a body placed at the center of earth, then

$$\text{Weight, } mg' = 0$$

Hence, the weight of the body at the center of earth is equal to zero.

Comparison of height and depth for the same change in 'g'

We know that the value of acceleration due to gravity decreases whenever one moves upward and also the value of acceleration due to gravity decreases whenever one moves downward. This indicates that the value of 'g' is maximum on the surface of earth.

$$g' = g\left(1 - \frac{2h}{R}\right)$$

$$g' = g\left(1 - \frac{d}{R}\right)$$

Comparing these two equations, we get

$$1 - \frac{2h}{R} = 1 - \frac{d}{R}$$

$$\Rightarrow -\frac{2h}{R} = -\frac{d}{R}$$

$$\Rightarrow \boxed{d = 2h}$$

Therefore, the value of acceleration due to gravity at a height 'h' from the surface of earth is same as the value of acceleration due to gravity at a depth $d = 2h$.

Kepler's Laws of Planetary Motion $\Rightarrow$

On the basis of motion of planets Kepler formulated three laws of planetary motion which are as follows.

① 1st Law (Law of elliptical orbits) :-

Statement :-

A planet revolves around the sun in an elliptical orbit with sun situated at one of ~~the focus~~ its ~~of the ellipse~~ foci.

Explanation :-

Since the focus of an ellipse is not equidistant from the point on the orbit, the distance of planet from sun varies between a certain minimum and a maximum value. Rotation is the reason for a change of seasons from winter to summer and repetition of the seasons after one year.

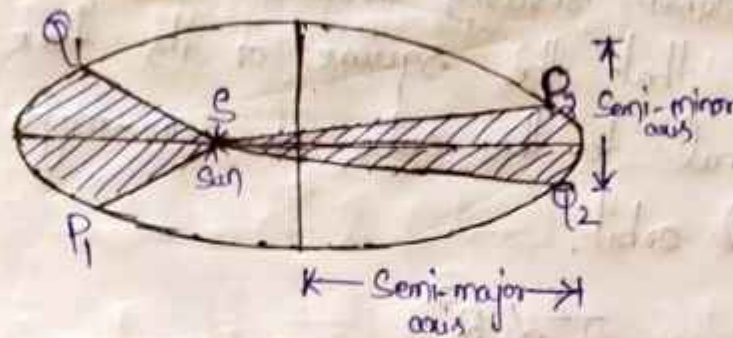
② 2nd Law (Law of areal velocities) :-

Statement :-

A planet revolves around the sun in such a way that its areal velocity is constant.

ie. the line joining the planet with the sun sweeps equal areas in equal interval of time.

Explanation →



Suppose the time taken by a planet to move from P_1 to Q_1 is same as the time taken by the planet to move from P_2 to Q_2

According to 2nd law,

$$\text{Area } P_1SQ_1 = \text{Area } P_2SQ_2$$

$$\Rightarrow \frac{1}{2} \underset{\text{height}}{SP_1} \times \underset{\text{base}}{P_1Q_1} = \frac{1}{2} \underset{\text{height}}{SP_2} \times \underset{\text{base}}{P_2Q_2} \quad (\text{for very small triangle})$$

$$\Rightarrow SP_1 \times P_1Q_1 = SP_2 \times P_2Q_2$$

$$\text{Since } SP_1 < SP_2$$

$$\therefore P_1Q_1 > P_2Q_2$$

Therefore the velocity with which the planet is revolving around sun (orbital velocity) is not same at all the position of the planet. It is maximum at a point which is nearer to sun but it is minimum at a point which is farther from the sun.

Due to this reason, the day time in Summer season (nearer position SP_1) is more than the day time in winter season (Farther position SP_2)

③ 3rd Law (Law of Time Periods or Harmonic Law)

Statement:-

A planet revolves around the sun in such a way that the square of its time period is proportional to the cube of semi-major axis of its elliptical orbit.

$$T^2 \propto R^3$$

If T_1 and T_2 are the time periods of two planets having R_1 and R_2 as the semi-major axes of the orbits of two planets,

then,

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Problems

- ① A body weights 36 newton on the surface of earth. Calculate the weight at a height equal to half the radius of earth. (Ans: 16N)
- ② A body weights 72 kgwt on the surface of earth. How much will it weight on the surface of mars whose mass is $\frac{1}{9}$ and radius is $\frac{1}{2}$ that of earth. (Ans: 32kgwt)
- ③ If velocity of a body is doubled and mass is halved, what happens to the kinetic energy. (Ans: 2KE)
- ④ What will be the change in kinetic energy when the momentum is doubled? (4KE)

⑤ At what height from the surface of earth, the value of 'g' will be reduced by 36% of the value at the surface? Radius of earth = 6400 km
(Ans: 1600 km)

Ans: Let at a height 'h', the value of 'g' reduces by 36% i.e. it becomes 64% of that at the surface.

$$\text{So, } g' = 64\% \text{ of } g \quad h = ?$$

$$= 0.64g \quad R = 6400 \text{ km}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{0.64g}{g} = \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{64}{100} = \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{8}{10} = \frac{R}{R+h}$$

$$\Rightarrow 8R + 8h = 10R$$

$$\Rightarrow 8h = 2R$$

$$\Rightarrow h = \frac{2 \times 6400}{8}$$

$$= 1600 \text{ km}$$

Electrostatics

Electrostatics is the branch of physics which deals with the study of electrified bodies at rest.
(charged)

Charge

It is the source of electric field. A body becomes charged whenever it gains or loses some electrons.

Properties of charge

- charge is a scalar quantity
- charge is quantized. i.e. charge on any body will be an integral multiple of e (elementary charge) i.e.
 $q = \pm ne$, where, $n = 1, 2, 3, \dots$ etc.
- charge can neither be created nor destroyed.
- charge on 1 proton/electron = 1.6×10^{-19} Coulomb.
- Like or similar charges repel and unlike or opposite charges attract each other.
- No. of electrons present in one coulomb of charge -

$$n = \frac{q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

Coulomb's Law in Electrostatics

Unit of charge

SI $\Rightarrow$ Coulomb

C.G.S $\Rightarrow$ Electrostatic unit $\rightarrow$ Stat-Coulomb
Electromagnetic unit $\rightarrow$ ab-Coulomb

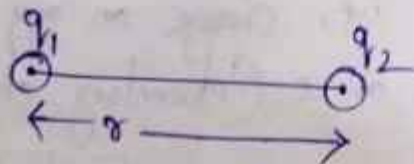
$$1 \text{ Coulomb} = \frac{1}{10} \text{ ab-Coulomb} \\ = 3 \times 10^9 \text{ Stat-Coulomb}$$

Coulomb's Law in Electrostatics:

Statement $\Rightarrow$

Point charge $\Rightarrow$ A body having negligible dimensions is termed as a point-charge. Such a body having a charge upon it is known as a point charge.

It states that the force of attraction or repulsion in between any two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance in between the two charges.



Suppose two point-charges q_1 & q_2 separated by a distance 'r'.

Then according to Coulomb's law $F \propto q_1 q_2$
 $\propto \frac{1}{r^2}$

$$\Rightarrow \boxed{F = \beta \frac{q_1 q_2}{r^2}} \quad \text{--- (1)}$$

Where β is a constant of proportionality whose value depends upon the medium in between the two charges and the system of unit chosen.

Value of β

① In C.G.S. System:

In C.G.S. electrostatics system of units (e.s.u.)

$$\beta = \frac{1}{k}$$

where k = Dielectric Constant of the medium

Then Coulomb's law in C.G.S. e.s.u. $\Rightarrow F = \frac{1}{k} \frac{q_1 q_2}{r^2}$ (medium) ②

For free space (Vacuum) $\Rightarrow k=1$

Coulomb's law $\Rightarrow F = \frac{q_1 q_2}{r^2}$ (air) ③

② S.I. System :-

In S.I. System $\Rightarrow \epsilon = \frac{1}{4\pi\epsilon}$

where ϵ = Absolute Permittivity of the medium

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\epsilon = \epsilon_0 \epsilon_r$$
 ④

where ϵ_0 = Absolute Permittivity of the free space

ϵ_r = Relative Permittivity of the medium.

Coulomb's law $\Rightarrow F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$

$$\Rightarrow F = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$
 (medium) ⑤

For free space (Vacuum) $\Rightarrow \epsilon_r = 1$

$$\Rightarrow \epsilon = \epsilon_0$$

Now Coulomb's law becomes $\Rightarrow F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ (air) ⑥

~~Value of ϵ_0~~

$$\text{Value of } \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$\text{Value of } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

Dimension of ϵ_0

$$\text{We know that } F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} = \frac{F r^2}{q_1 q_2}$$

$$\Rightarrow 4\pi\epsilon_0 = \frac{q_1 q_2}{Fr^2}$$

$$\begin{aligned}\epsilon_0 &= \frac{q_1 q_2}{Fr^2 4\pi} \\ &= \frac{[A^2 T^2]}{[M^1 L^1 T^{-2}] \times [L^2]}\end{aligned}$$

$$[\epsilon_0] = [M^{-1} L^{-3} T^4 A^2]$$

Unit charge:

Unit charge is a charge of that much strength which when placed at a distance of 1m from a similar charge repels it with a force of $\frac{1}{4\pi\epsilon_0}$ Newton or 9×10^9 Newton.

Electric Field

Electric field is the region in space surrounding a charge in which if any other charge is brought in that will experience a force.

Electric Field Intensity ($\vec{E}$)

Electric field intensity at any point inside the electric field is defined as the amount of force experienced by a unit test charge placed at that point.
(+ve charge)

$$\text{Mathematically, } \vec{E} = \frac{\vec{F}}{+q}$$

Where E = Electric field intensity
 F = Electrostatic force
 $+q$ = Test charge

Electric field intensity is a vector quantity. It is directed away from the charge if the source charge is +ve. But it is directed towards the charge if the source charge is negative.

Unit

① C.G.S. System $\Rightarrow$ Dyne/Stat-Coulomb

② S.I. System $\Rightarrow$ Newton/Coulomb

Dimension

$$[E] = \frac{[F]}{[q]} = \frac{[M^1 L^1 T^{-2}]}{[A^1 T^1]}$$

$$[E] = [M^1 L T^{-2} A^{-1}]$$

Electric potential (V)

$\rightarrow$ Electric potential is a quantity which determines the direction of flow of charge in a field.
 $\rightarrow$ Electric potential at any point inside the electric field is defined as the amount of work done in moving a unit positive charge (test charge) from infinity to that point against the electric field.

Rule

If the source charge is positive, unit positive charge is moved from infinity to that point while in case of negative source charge, unit positive charge is taken from the given point to infinity.

If W is the amount of work done in moving a test charge $+q$ from infinity to the point at which the electric potential is to be determined.

Then electric potential $V = \frac{W}{+q}$

→ Electric potential (V) is a scalar quantity.

Unit of V

→ (1) S.I. System $\Rightarrow \frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$

(2) C.G.S. System $\Rightarrow \frac{\text{Erg}}{\text{Stat Coulomb}} = \text{Stat-Volt}$

$1 \text{ Stat-Volt} = 300 \text{ Volt}$

Dimension $\Rightarrow V = \frac{W}{q} = \frac{\text{Force} \times \text{displacement}}{\text{Current} \times \text{Time}}$

$\Rightarrow [V] = \frac{[M^1 L^2 T^{-2}]}{[A^1 T^1]} = [M^1 L^2 T^{-3} A^{-1}]$

Note

$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

Potential is +ve for a +ve source & -ve for a -ve source charge.

Capacity or Capacitance of a Conductor

The Capacity of a conductor is defined as the ratio between the charge on the conductor to its potential.

Mathematically, Capacity (C) = $\frac{Q}{V}$

Where Q = charge

V = electric potential

Unit

S.I. unit $\Rightarrow$ Farad

C.G.S. Unit $\Rightarrow$ Stat-Farad

~~1 Farad~~ $1 \text{ Farad} = 9 \times 10^{11} \text{ Stat-Farad}$

Dimension $\Rightarrow [C] = [M^{-1}L^{-2}T^4A^2]$

Capacitor :-

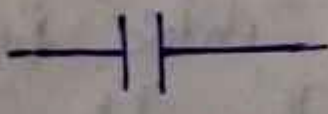
$\rightarrow$ A ~~com~~ combination of two conductors ~~separated~~ separated by a fixed distance with an insulating or dielectric medium in between is called Capacitor or Condenser.

$\rightarrow$ The two conductors of the capacitor have charges equal in magnitude and opposite sign. Thus the net charge on the capacitor as a whole remains zero.

Ex:- ① A Spherical Capacitor (2 Concentric spheres)

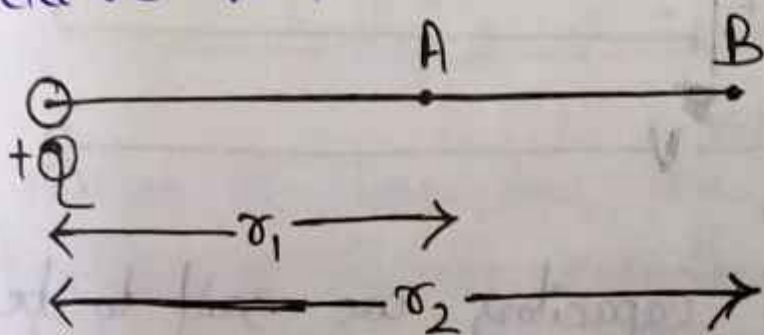
② A cylindrical Capacitor (2 co-axial cylinders)

③ A parallel plate Capacitor

$\rightarrow$ Circuit-Symbol $\Rightarrow$ 

Electric Potential difference $\Rightarrow$

Electric potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge (test charge) from one point to the other against the electric field.



Let us consider A and B are two points which are situated at distances r_1 and r_2 respectively from the charge $+Q$.

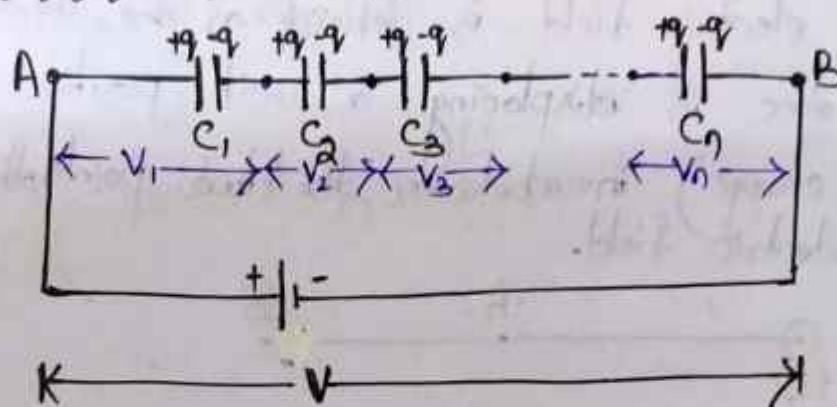
$$V_A = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_1}$$

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_2}$$

$$V_A - V_B = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Grouping of Capacitor $\Rightarrow$

(a) Capacitors in Series $\Rightarrow$



A number of capacitors are said to be connected in a series if the 2nd plate of the 1st capacitor is connected with the 1st plate of the 2nd capacitor and so on.

Suppose 'n' capacitors having capacities $C_1, C_2, C_3, \dots, C_n$ are connected in series.

In series connection, the charge on each capacitor is same. So the potential difference between their plates are different.

Let $V_1, V_2, V_3, \dots, V_n$ are the potential difference between the plates of the capacitors.

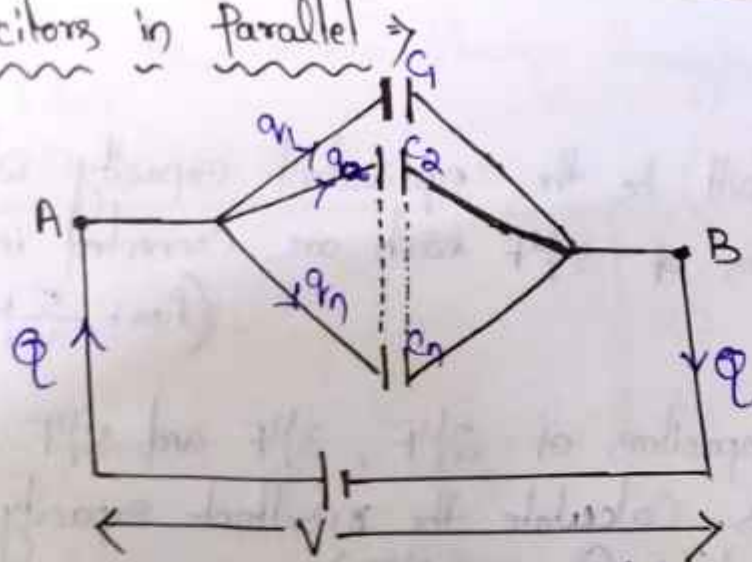
$$\text{Then } V = V_1 + V_2 + V_3 + \dots + V_n$$

$$\Rightarrow \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + \dots + \frac{q}{C_n}$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}}$$

Therefore, when a no. of capacitors are connected in series, the reciprocal of the resultant capacity is equal to the sum of the reciprocal of their individual capacity.

(b) Capacitors in Parallel



A number of capacitors are ^{said to be} connected in parallel if 1st plate of all the capacitors are connected together at a point and 2nd plate of all the capacitor are also connected together at ~~a~~ point. another point.

In parallel grouping the potential difference between the plates of the capacitors are same. But their charges are different.

Let 'n' number of capacitors having capacity $C_1, C_2, C_3, \dots, C_n$ are connected in parallel and their charges are $q_1, q_2, q_3, \dots, q_n$ respectively.

If Q is the total charge drawn from the source, then

$$Q = q_1 + q_2 + q_3 + \dots + q_n$$

$$\Rightarrow CV = C_1V + C_2V + C_3V + \dots + C_nV$$

$$\Rightarrow \boxed{C = C_1 + C_2 + C_3 + \dots + C_n}$$

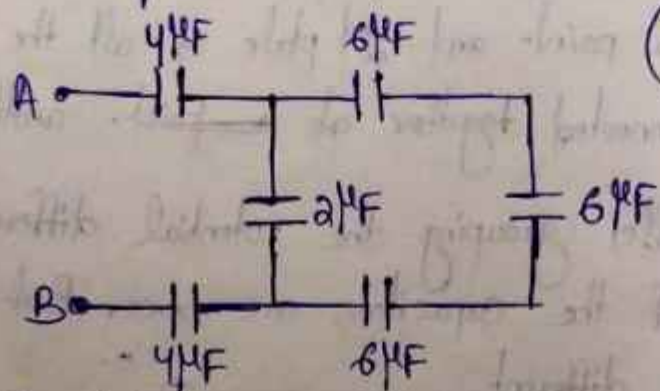
Therefore, the resultant capacity of a number of capacitors, connected in parallel is equal to the sum of their individual capacities.

Problems

1) What will be the equivalent capacity, when 4 capacitors of $5\mu\text{F}$ each are connected in series.
(Ans: $\frac{5}{4}\mu\text{F}$)

2) Three capacitors of $2\mu\text{F}$, $3\mu\text{F}$ and $5\mu\text{F}$ are connected in series. Calculate the resultant capacity of the combination. (Ans: $0.97\mu\text{F}$)

3) Find the equivalent capacitance between A and B.
(Ans: $\frac{4}{3}\mu\text{F}$)



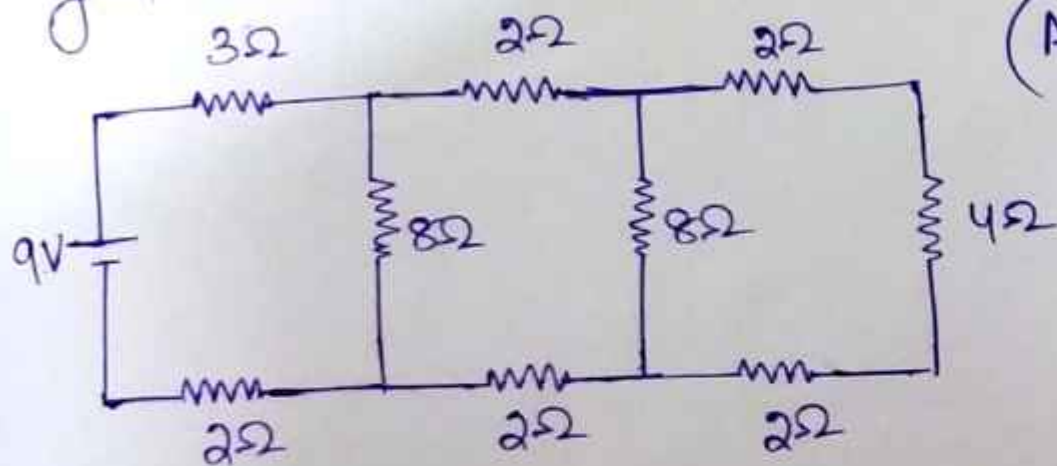
4) Find the total capacity when two capacitors of $100\mu\text{F}$ and 0.5mF are connected in series.
(Ans: $0.083 \times 10^{-3}\text{F}$)

5) What will be the equivalent resistances when 5 resistors of 10Ω each are connected in series and in parallel? (Ans: 50Ω & 2Ω)

6) What will be the equivalent resistance when 5 resistors of resistance 3Ω each are connected in parallel. (Ans: $\frac{3}{5}\Omega$)

7) Find the charge on a capacitor of capacity $50\mu\text{F}$ having a potential of 200V . (Ans: 10^{-2}C)

8) Calculate the current flowing through the circuit given below:



(Ans: $I = 1A$)

9) ~~What will be the equivalent resistances when 5 resistors of~~

Magnetism

Magnetization

A piece of substance which possesses the property of attracting small pieces of iron towards it, is called a magnet.

Natural Magnet $\rightarrow$ Magnetite / lodestone
(Fe_3O_4 oxide iron)

Properties of a Magnet $\rightarrow$ Artificial Magnet $\rightarrow$ Iron, Nickel & Cobalt by rubbing with Fe_3O_4 oxide iron

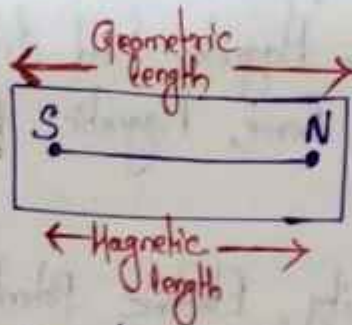
An magnet possesses following properties:

① Two Poles of a magnet:-

Every magnet has two poles north pole and South pole. The poles of a magnet are situated near the two ends of a magnet. The distance between the two poles is known as Magnetic length. But the distance between the two

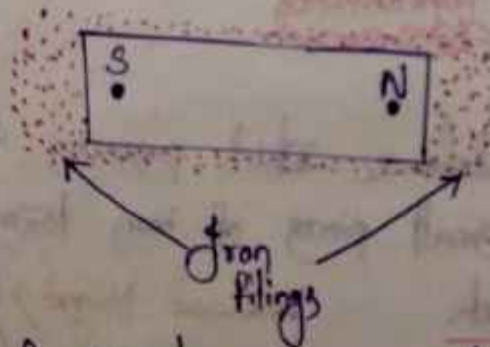
ends of a magnet is known as Geometric Length.

$$\text{Magnetic Length} = \frac{7}{8} \times \text{Geometric length}$$



② Attractive property of a magnet :-

Every magnet attracts small pieces of iron towards it. The attracting power is maximum near the two poles, but minimum near the centre of the magnet.



③ Directional property of a magnet :-

When a bar magnet is freely suspended, then one end of the magnet points towards geographic north which is known as north pole. and the other end of the magnet points towards the geographic south which is known as South pole of the magnet.

④ No existence of isolated magnetic poles:
Magnetic poles always exist in pairs. Isolated magnetic pole can never exist.

⑤ Nature of the force between two poles:-
Like poles repel, but unlike poles attract each other.

Poles

→ Poles of the magnet are the points which show maximum magnetic property.

→ Strength of the two poles are always equal.

→ Unit of pole strength (m) = Amp. m or Weber.

Dimension $\Rightarrow$ pole strength (m) = $\frac{\text{force}}{\text{magnetic flux density}} = \frac{[N^1 L^1 T^{-2}]}{[N^1 L^0 T^{-2} A^{-1}]} = \frac{[N^1 L^1 T^{-2}]}{[N^1 L^0 T^{-2} A^{-1}]}$

Coulomb's Law in Magnetism $\Rightarrow [m] = [L^1 A^1]$

$1 \text{ tesla} = \frac{Wb}{m^2}$

* Force of attraction or repulsion between two poles obeys inverse square law.

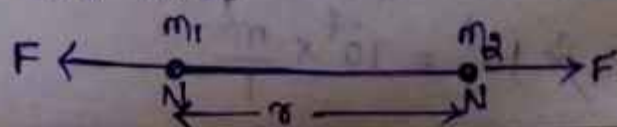
* Magnetic force between two poles is a central force.

Statement:-

The force of attraction or repulsion in between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between the two poles.

Explanation:-

Consider two magnetic poles of similar nature of strengths m_1 and m_2 separated a distance r from each other.



Force of repulsion between them depends upon following factors.

$$F \propto m_1 m_2$$

$$\propto \frac{1}{r^2}$$

$$\Rightarrow F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow F = k \frac{m_1 m_2}{r^2}$$

Where 'k' is the constant of proportionality.

In S.I. system $\Rightarrow k = \frac{\mu_0}{4\pi} \Rightarrow F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$

$$k = \frac{\mu}{4\pi}$$

μ = Permeability of medium between the poles

$$F = \frac{\mu}{4\pi} \frac{m_1 m_2}{r^2}$$

$$\mu = \mu_0 \mu_r$$

μ_0 is called the "absolute magnetic permeability" of free space.

In C.G.S. system $\Rightarrow k=1 \Rightarrow F = \frac{m_1 m_2}{r^2}$

Unit Pole

① In S.I. system:

$$[\mu_0] = \frac{\text{force} \times (\text{distance})^2}{(\text{pole strength})^2} = \frac{[N] [L]^2}{[A]^2} = \frac{[N] [L]^2 [T^{-2}]}{[A]^2} = [N] [L] [T^{-2}] [A^{-2}]$$

According to Coulomb's law, in S.I.

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

Since $\mu_0 = 4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1}$

$$F = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{m_1 m_2}{r^2}$$

$$\Rightarrow F = 10^{-7} \frac{m_1 m_2}{r^2}$$

If $F = 10^{-7} \text{ N}$, $m_1 = m_2 = m$ & $r = 1 \text{ m}$

$$\Rightarrow 10^{-7} = 10^{-7} \times \frac{m^2}{1}$$

$$\begin{aligned} \Phi_B &= BA \\ 1 \text{ Weber} &= 1 \text{ Tesla} \times 1 \text{ m}^2 \\ B &= \frac{F}{q \times v} = \frac{F}{1 \times \frac{1}{s}} = \frac{F}{1 \times s} = \frac{N \cdot A^{-1} \cdot m^{-1}}{1 \times s} = 1 \text{ Tesla} \\ 1 \text{ Weber} &= \frac{N}{\text{Amp} \cdot \text{m}} \times \text{m}^2 = \frac{1 \text{ N} \cdot \text{m}}{\text{Amp}} \\ \mu_0 &= \frac{N}{A^2} = \frac{N \cdot \text{m}}{A^2 \cdot \text{m}} = \text{Wb A}^{-1} \text{ m}^{-1} \end{aligned}$$

S.I. $\Rightarrow \mu_0$ = Permeability of free space

μ_r = Relative Permeability of the medium

$$F = \frac{\mu_0 \mu_r}{4\pi} \frac{m_1 m_2}{r^2}$$

When the medium between the poles is air $\Rightarrow \mu_r = 1 \Rightarrow \mu = \mu_0$

$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

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$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

$$[\mu_0] = [N] [L] [T^{-2}] [A^{-2}]$$

$$\Rightarrow m^2 = 1$$

$$\Rightarrow m = \pm 1 \quad (\text{unit-pole})$$

A unit pole, in S.I. is that pole which when placed in air at a distance of 1m from a similar pole repels it with a force of 10^{-4} N.

② In C.G.S. system:-

According to Coulomb's law in C.G.S system

$$F = \frac{m_1 m_2}{r^2}$$

If $F = 1 \text{ dyne}$, $m_1 = m_2 = m$ and $r = 1 \text{ cm}$

$$1 = \frac{m^2}{1}$$

$$\Rightarrow m^2 = 1$$

$$\Rightarrow m = \pm 1$$

A unit pole, in C.G.S. system, is that pole which when placed in air at a distance of 1cm from a similar pole repels it with a force of 1 dyne.

Magnetic Field:-

Magnetic field, of any magnetic pole, is the region (space) around it in which its magnetic influence can be realized.

Magnetic Field Intensity (H)

Magnetic field intensity at any point inside the magnetic field is defined as the amount of force experienced by a unit north pole placed at that point.

* The direction of magnetic field intensity is the direction in which the unit north pole would move if it were free to do so.

→ Magnetic intensity, at any point, is a vector quantity.

(i) In S.I. system

Mathematically, $\vec{H} = \frac{\vec{F}}{m_0}$

where,

$\vec{H}$ = Magnetic field intensity

$\vec{F}$ = Magnetostatic force

m_0 = Pole strength of test pole

Force between two poles is,

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

If $m_1 = m$, $m_2 = 1$

$$\Rightarrow F = \frac{\mu_0}{4\pi} \frac{m}{r^2}$$

$$\Rightarrow \boxed{H = \frac{\mu_0}{4\pi} \frac{m}{r^2}}$$



S.I. unit $\rightarrow N/A \cdot m$

$$= N A^{-1} m^{-1}$$

$$\text{Dimension} \rightarrow [H] = \frac{[M L T^{-2}]}{[A]} = [M L T^{-2} A^{-1}]$$

(ii) In C.G.S. system

$$F = \frac{m_1 m_2}{r^2}$$

If $m_1 = m$, $m_2 = 1$

$$\Rightarrow F = \frac{m}{r^2}$$

$$\Rightarrow \boxed{H = \frac{m}{r^2}}$$

Magnetic lines of force:

Magnetic lines of force is the path along which a unit north pole would move if it were free to do so.

OR

These are imaginary closed curves drawn in a magnetic field such that the tangent drawn at any point of the curve gives the direction of resultant magnetic field at that point.



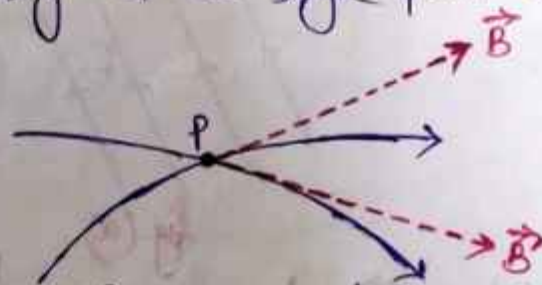
(fig-a)

Properties of Magnetic lines of Force:

- (1) Magnetic lines of force are always directed away from the north pole and towards the south pole, or from north to south outside the magnet and south to north inside the magnet.
- (2) Tangent, at any point, to the magnetic line of force gives the direction of magnetic intensity at the point.



- (3) Two magnetic lines of force can never intersect each other. If they intersect, then, at a single point two tangents can be drawn which gives two directions of the magnetic field intensity at a single point which is impossible.



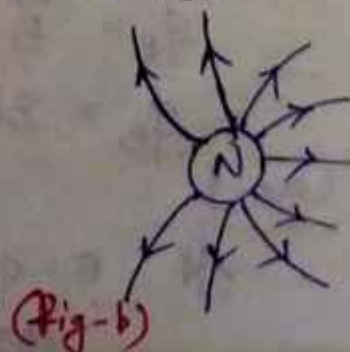
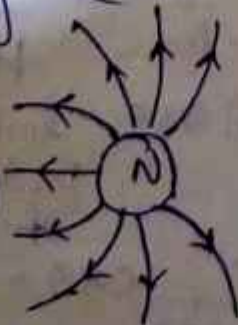
- (4) The number of lines of force per unit area (Area being perpendicular to lines) is proportional to magnitude of ~~strength~~ magnetic field intensity at that point.

Thus, more concentration of lines represents stronger magnetic field.

- (5) The lines of force ~~can~~ tend to contract longitudinally or lengthwise i.e., they possess longitudinal strain.

Due to this property the two unlike poles attract each other. (Fig-a)

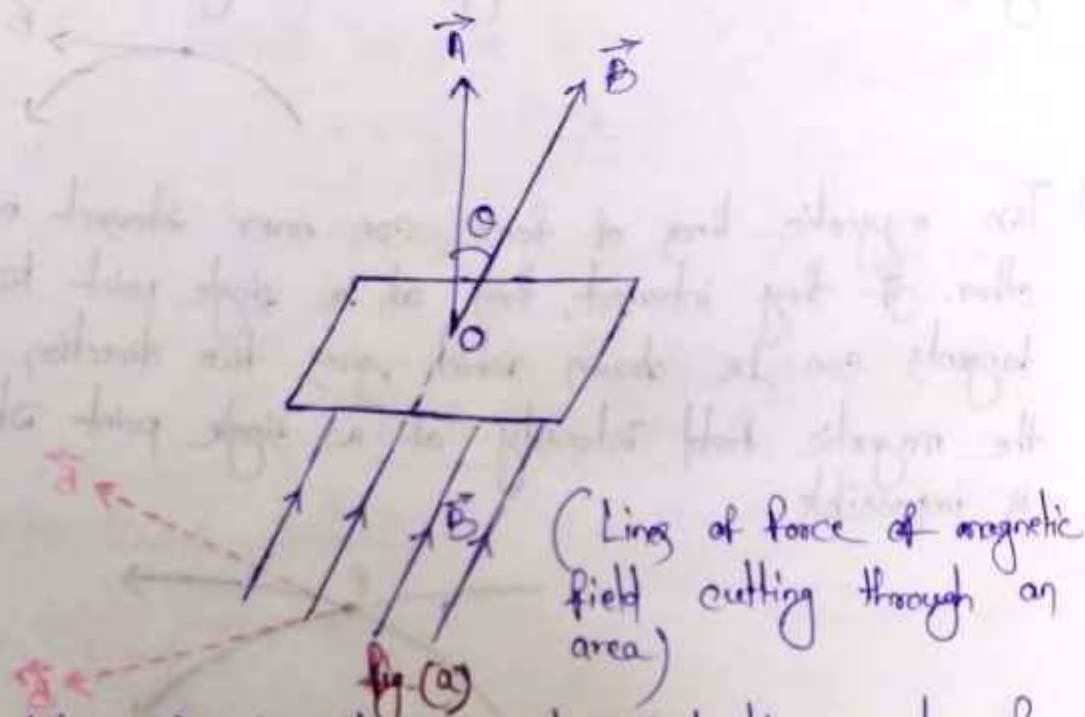
- (6) The lines of force tend to exert lateral (sideways) pressure, i.e., they repel each other laterally.



This explains the repulsion between two similar poles. (Fig-b)

Magnetic Flux: Magnetic flux deals with the study of the number of lines of force of magnetic field crossing a certain area.

Magnetic flux linked with a surface is the dot product of magnetic induction vector and Area vector.



Let $\vec{B}$ be the magnetic induction vector & $\vec{A}$ be the area vector directed along the normal to the surface, and having a length proportional to the magnitude of the area. The angle betⁿ $\vec{B}$ and $\vec{A}$ is θ .

Then the Magnetic flux linked through the surface is given by

$$\Phi_B = \vec{B} \cdot \vec{A}$$

$$\Phi_B = BA \cos \theta$$

Case-1

If $\theta = 0^\circ$, $\cos \theta = 1$

$$\Phi_B = BA$$

Defⁿ Magnetic flux linked with a surface, is defined as the product of area & the component of 'B' perpendicular to the area.

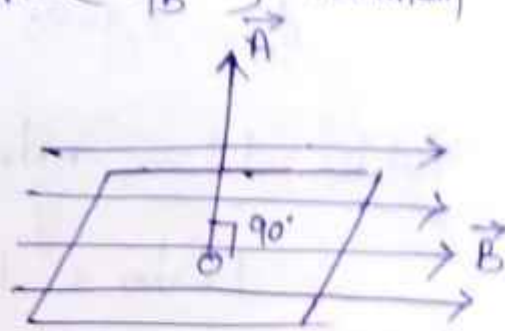
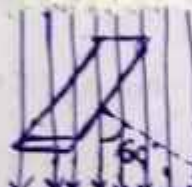
Therefore, Magnetic flux ϕ_B is maximum at $\theta = 0^\circ$

Case-2

If $\theta = 60^\circ$, $\cos 60^\circ = \frac{1}{2}$

$$\phi_B = BA \cos 60^\circ$$

$$\phi_B = \frac{BA}{2}$$



(Surface held parallel to lines of force)

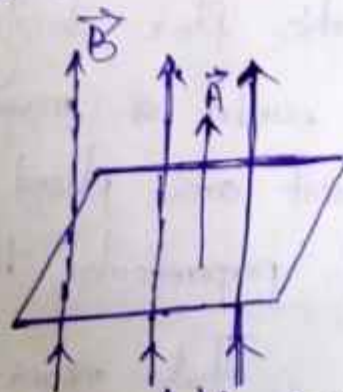
(Case-3)

Case-2

If $\theta = 90^\circ$, $\cos \theta = 0$
 $\phi_B = 0$

(Normal to the surface held at 60° to the lines of force)

Therefore, Magnetic flux ϕ_B is minimum at $\theta = 90^\circ$.



(Surface held perpendicular to lines of force)

(Case-1)

Units of Magnetic Flux:

a) S.I. unit $\Rightarrow$ Weber (Wb)

$$\phi_B = BA$$

$$B = 1 \text{ tesla (T)}$$

$$A = 1 \text{ m}^2$$

$$\phi_B = 1 \text{ tesla} \times 1 \text{ m}^2 = 1 \text{ Weber}$$

Dimensional formula of ϕ_B :

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$

$$\phi_B = \frac{F}{qV} A \cos \theta$$

$$[\phi_B] = \frac{[M^1 L^1 T^{-2}] \times [L^2]}{[A^1 T^{-1}] [L T^{-1}]}$$

$$[\phi_B] = [M^1 L^2 T^{-2} A^{-1}]$$

b) C.G.S. Unit $\Rightarrow$ Maxwell

$$B = 1 \text{ gauss (G)}$$

$$A = 1 \text{ cm}^2$$

$$\phi_B = 1 \text{ gauss} \times 1 \text{ cm}^2 = 1 \text{ Maxwell}$$

Relation between Weber & Maxwell :-

$$\begin{aligned} 1 \text{ Weber} &= 1 \text{ Tesla} \times 1 \text{ m}^2 \\ &= 10^4 \text{ gauss} \times (100 \text{ cm})^2 \\ &= 10^8 \text{ gauss cm}^2 \end{aligned}$$

$$\boxed{1 \text{ Weber} = 10^8 \text{ Maxwell}}$$

Magnetic Flux Density / Magnetic Induction ($\vec{B}$)

Magnetic flux density, at any point, is defined as the number of magnetic lines of force passing through a unit area placed at that point - if the area is held perpendicular to the direction of lines.

That means $B = \frac{\phi_B}{A}$ figure $\Rightarrow$ Here $\theta = 0^\circ$

Units

S.I. System $\Rightarrow$ Tesla (T) / $\frac{\text{Weber/m}^2}{(\text{Wb m}^{-2})}$ / $\text{N A}^{-1} \text{ m}^{-1}$

C.G.S. System $\Rightarrow$ Gauss (G) / $\frac{\text{Maxwell}}{\text{cm}^2}$

Dimension $\Rightarrow [B] = [M^1 L^0 T^{-2} A^{-1}]$

* Magnetic Permeability

Permeability is the measure of the ability of a material to support the formation of a magnetic field within itself.

$$\begin{aligned} [B] &= \frac{|\vec{F}_{\text{max}}|}{|q| |\vec{v}|} \\ &= \frac{[M^1 L^0 T^{-2}]}{[A^1 T^1] \times [L^1 T^{-1}]} \\ &= [M^1 L^0 T^{-2} A^{-1}] \end{aligned}$$

$$\vec{F} = q(\vec{v} \times \vec{B})$$